Single transverse-spin asymmetry in inclusive hadron production

Zhong-Bo Kang

RIKEN BNL Research Center
Brookhaven National Laboratory

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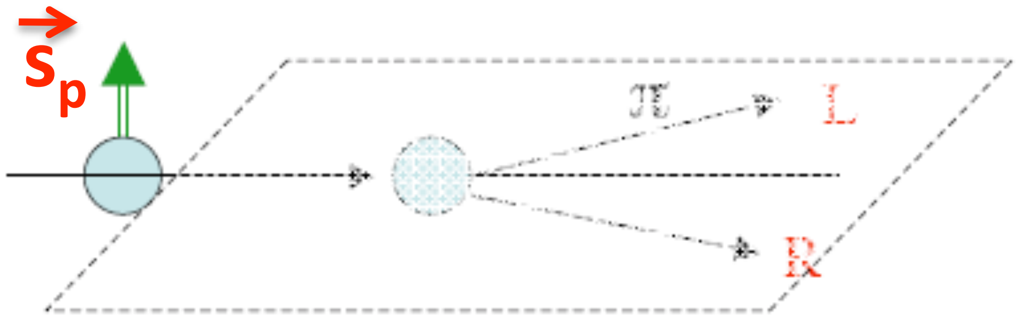
based on work with Gamberg ,Qiu, Vogelsang, Yuan, Zhou

Outline

- Introduction
- Collinear factorization approach:
  - Basic formalism
  - Quark and gluon contributions
  - pT behavior
  - Global analysis
  - Twist-3 fragmentation function contribution
- TMD approach:
  - Generalized Parton Model (GPM)
  - Process-dependent Sivers function
- Summary

A_N Definition: Single Transverse Spin Asymmetry (SSA)

- Consider the scattering of a transversely polarized nucleon with another nucleon, observe a particle going left or right: left-right asymmetry



$$A_N = \frac{N_L - N_R}{N_L + N_R}$$

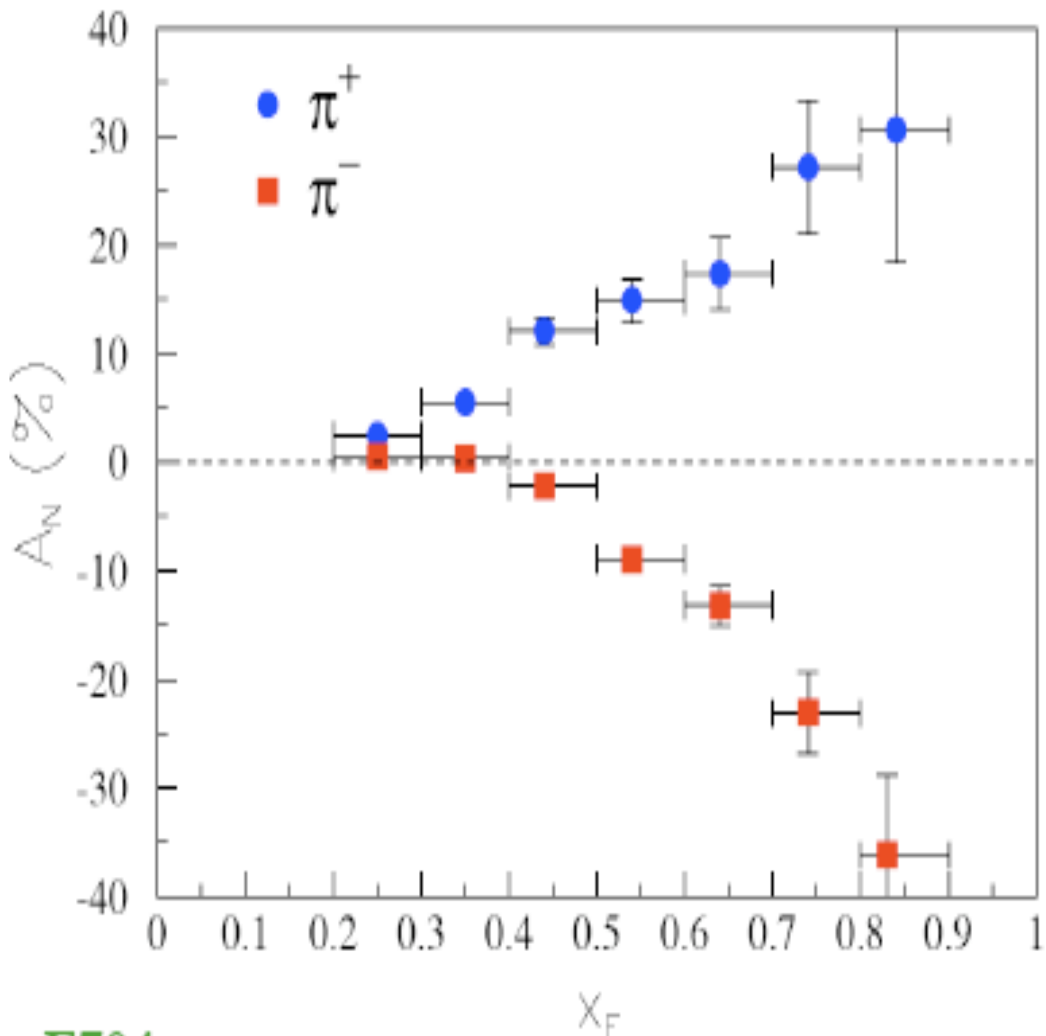
- Because of rotational symmetry, this corresponds to an asymmetry relate to the difference of the cross section when the spin of the incoming nucleon is flipped
  - Spin-averaged cross section: $\sigma(\ell) = \frac{1}{2}\left[\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})\right]$
  - Spin-dependent cross section: $\Delta\sigma(\ell, \vec{s}) = \frac{1}{2}\left[\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})\right]$
  - Single transverse-spin asymmetry (SSA):

$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

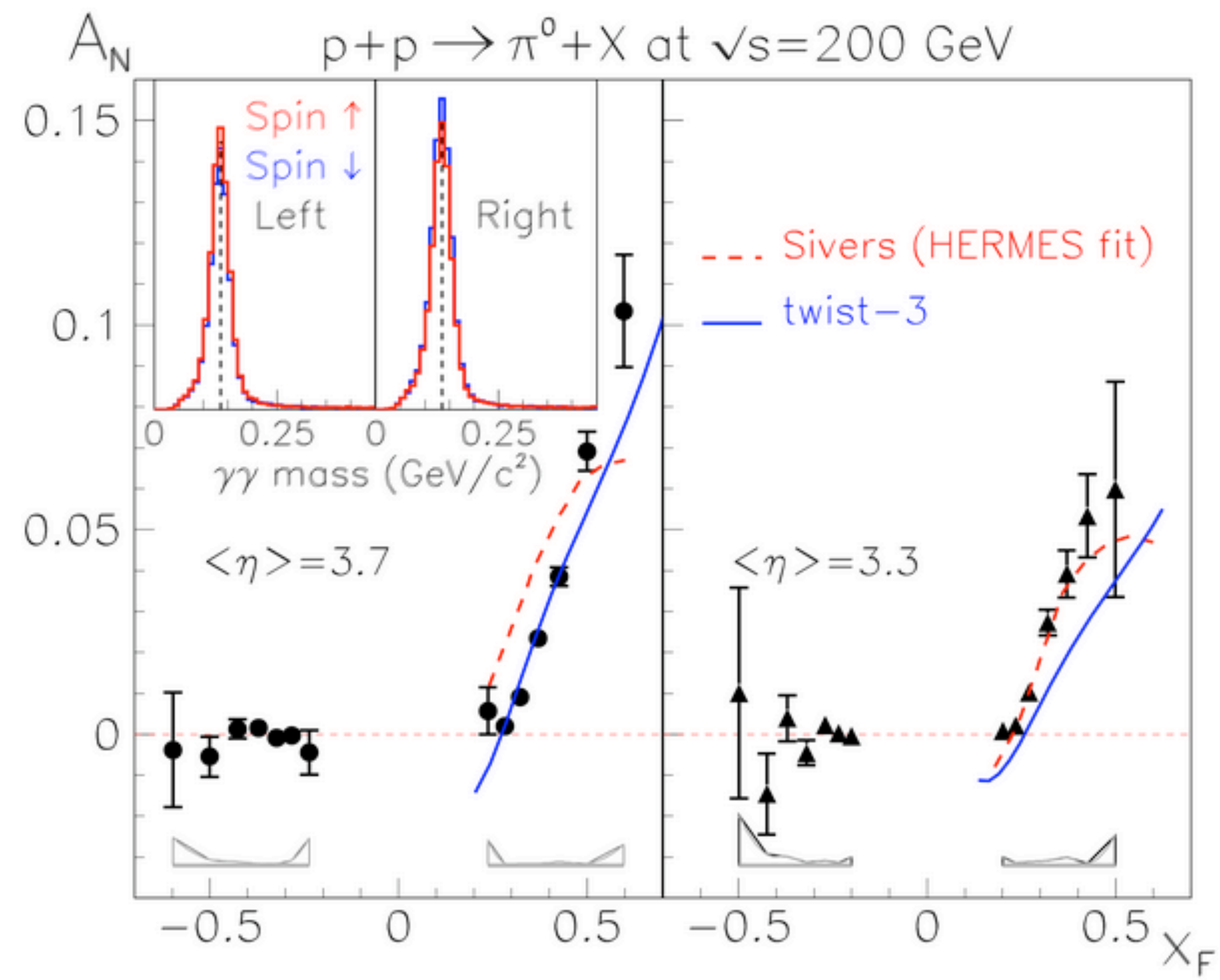
Experiment: Single Spin Asymmetries

- Fermilab E704, STAR, PHENIX, BRAHMS, COMPASS, HERMES, JLAB:

$$p^{\uparrow}p \to \pi X$$



E704



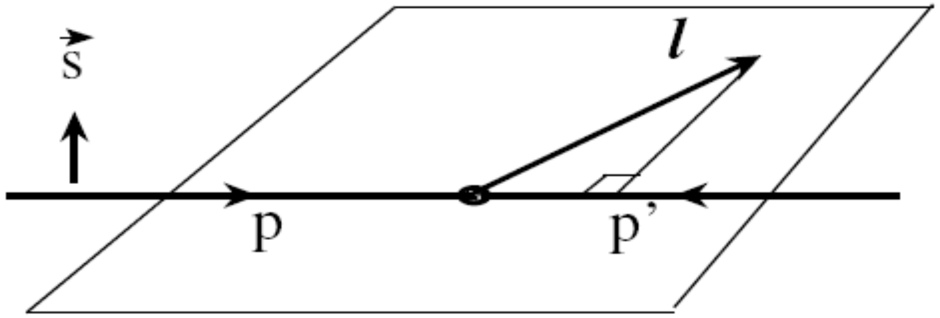
STAR

SSAs are observed in various experiments at different √s

SSA corresponds to a T-odd triplet product

- SSA measures the correlation between the hadron spin and the production plane, which corresponds to $\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$

$$p^{\uparrow} p \to \pi(\ell) X$$



- Such a product is (naive) odd under time reversal (T-odd), thus they can arise in a time-reversal invariant theory (eg, QCD) only when there is a phase between different spin amplitudes

$$\Rightarrow \quad A_N \propto i \vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$

  - the phase "i" is required by time-reversal invariance
  - covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$

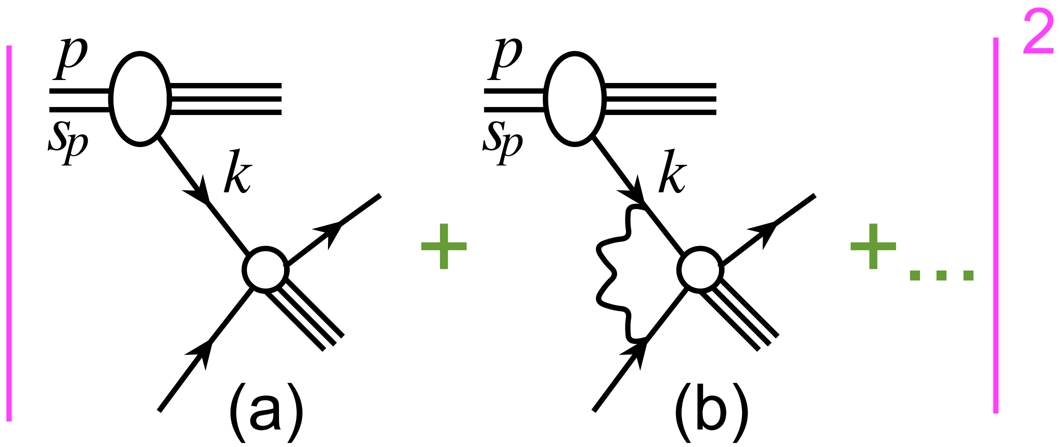
Nonvanishing A_N requires a phase, a helicity flip, and enough vectors to fix a scattering plane

SSA vanishes at leading twist in collinear factorization

- At leading twist formalism: partons are collinear

Kane, Pumplin, Repko, 1978

$\sigma(s_T) \sim$



➡ $\Delta\sigma(s_T) \sim \mathrm{Re}[(a)]\cdot\mathrm{Im}[(b)]$

  - generate phase from loop diagrams, proportional to α_s
  - helicity is conserved for massless partons, helicity-flip is proportional to current quark mass m_q

Therefore we have

$$A_N \sim \alpha_s \frac{m_q}{\sqrt{s}} \to 0$$

- $A_N \neq 0$: result of parton's transverse motion or correlations!

Two mechanisms to generate SSA in QCD

- SSA is related to parton's transverse motion
- TMD approach: Transverse Momentum Dependent distributions probe the parton's intrinsic transverse momentum
  - Sivers function: in Parton Distribution Function (PDF)

    Sivers 90
  - Collins function: in Fragmentation Function (FF)

    Collins 93
- Collinear factorization approach:
  - Twist-3 three-parton correlation functions: Qiu-Sterman matrix element, ...

    Efremov-Teryaev 82, 84, Qiu-Sterman 91, 98, ...
  - Twist-3 three-parton fragmentation functions:

    Koike, 02, Zhou, Yuan, 09, Kang, Yuan, Zhou, 10

Relation between twist-3 and TMD approaches

- They apply in different kinematic domain:
  - TMD approach: need TMD factorization, applies for the process with two observed momentum scales: DY at small q_T

$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse momentum} \end{cases}$$

  - Collinear factorization approach: more relevant for single scale hard process: inclusive pion production at pp collision
- They generate same results in the overlap region when they both apply:
  - Twist-3 three-parton correlation in distribution ⟷ Sivers function

    Ji, Qiu, Vogelsang, Yuan, 06, ...
  - Twist-3 three-parton correlation in fragmentation ⟷ Collins function

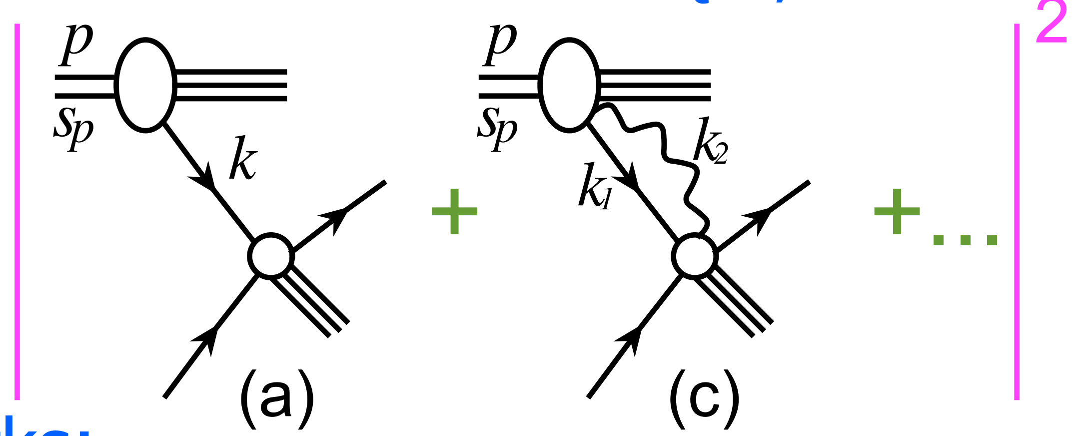
    Zhou, Yuan, 09, Kang, Yuan, Zhou, 10

SSA in collinear factorization approach

Efremov-Teryaev, 1982, Qiu-Sterman, 1991

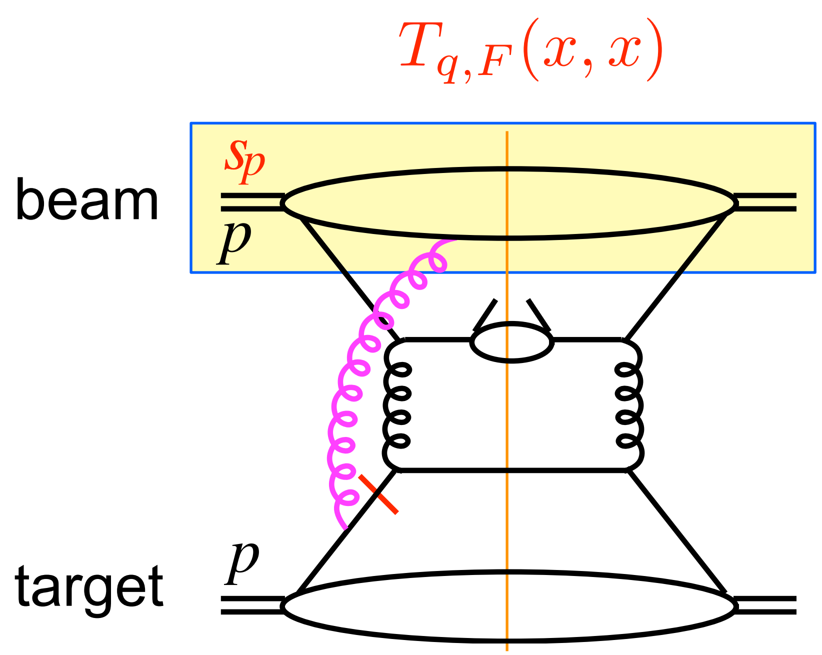
- When all observed scales >> Λ_{QCD}, collinear factorization should work:

$\sigma(s_T) \sim$



$\Rightarrow \Delta\sigma(s_T) \sim \mathrm{Re}[(a)]\cdot\mathrm{Im}[(c)]$

- How it works:



some propagators in the tree diagrams go on-shell

$$\frac{1}{k^2 - m^2 + i\epsilon} = P\frac{1}{k^2 - m^2} - i\pi\delta(k^2 - m^2)$$

- phase: from hard scattering amplitudes (unpinched pole)

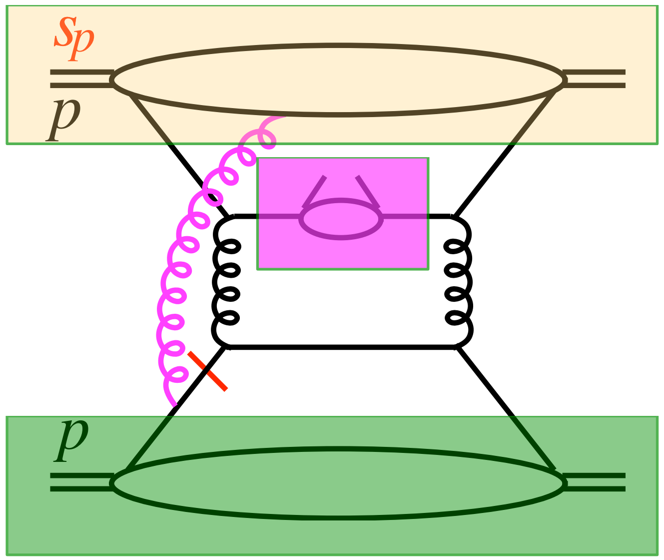
- spin flip: from interference between a quark state and a quark-gluon composite state

- Twist-3 quark-gluon correlation function $T_{q,F}(x,x)$:

$$T_{q,F}(x,x) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ixP^+y_1^-} \langle P, s_T | \bar{\psi}_q(0)\gamma^+ \boxed{\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma^{\ +}(y_2^-)\right]} \psi_q(y_1^-) | P, s_T \rangle$$

The sources of the twist-3 effects

- Three places where the twist-3 effects could come from:



- From polarized hadron
- From unpolarized hadron
- From fragmentation function

$$\Delta\sigma_{A+B\to hX}(\ell_\perp, \vec{s}_T) = \sum_{abc} \phi^{(3)}_{a/A}(x_1, x_2, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H_{ab\to c}(\ell_\perp, \vec{s}_T) \otimes D_{c\to h}(z)$$
$$+ \sum_{abc} \delta q_{a/A}(x, \vec{s}_T) \otimes \phi^{(3)}_{b/B}(x'_1, x'_2) \otimes H'_{ab\to c}(\ell_\perp, \vec{s}_T) \otimes D_{c\to h}(z)$$
$$+ \sum_{abc} \delta q_{a/A}(x, \vec{s}_T) \otimes \phi_{b/B}(x') \otimes H''_{ab\to c}(\ell_\perp, \vec{s}_T) \otimes D^{(3)}_{c\to h}(z_1, z_2)$$

- Within each term, there could be several twist-3 correlation functions

Twist-3 correlation function in polarized nucleon

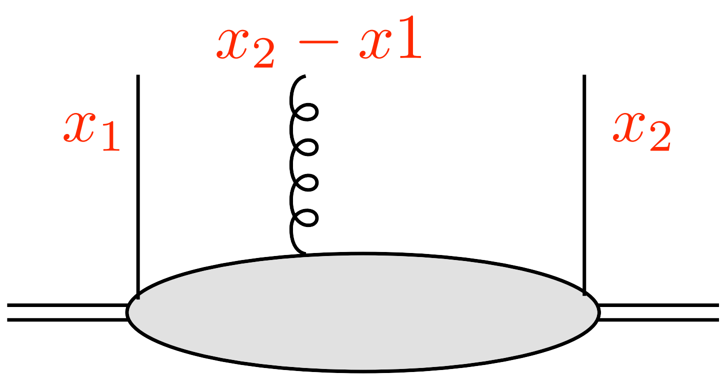
- quark-gluon correlation:

$$\mathcal{M}^\sigma(x_1,x_2) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ix_1p^+y_1^- + i(x_2-x_1)p^+y_2}\langle p,s_T|\bar\psi_q(0)gF^{\sigma+}(y_2^-)\psi_q(y_1^-)|p,s_T\rangle$$
$$= \frac{1}{2}\left[\not{p}\,\epsilon^{\sigma s_T n\bar{n}}T_{q,F}(x_1,x_2) + \gamma^5\not{p}\,is_T^\sigma T_{\Delta q,F}(x_1,x_2) + \cdots\right]$$

- Symmetry property:

$$T_{q,F}(x_1,x_2) = T_{q,F}(x_2,x_1)$$

$$T_{\Delta q,F}(x_1,x_2) = -T_{\Delta q,F}(x_2,x_1) \Rightarrow T_{\Delta q,F}(x,x) = 0$$



  - Soft gluonic pole: $T_{q,F}(x,x)$
  - Soft fermionc pole: $T_{q,F}(0,x), T_{\Delta q,F}(0,x)$

- Relation between $T_{q,F}(x,x)$ and quark Sivers $f_{1T}^\perp(x,k_\perp^2)$

Boer, Mulders, Pijlman, 2003

$$T_{q,F}(x,x) = \int d^2k_\perp \frac{|\vec{k}_\perp|^2}{M_h} f_{1T}^\perp(x,k_\perp^2)$$

Guidance for the relative size

- pQCD factorization theorem, the twist-3 correlation functions are universal but unknown, which need to be extracted from the experimental data
  - Or lattice: see P. Haegler's talk (July 21)
- With limited data and too many correlation functions, hopefully one could start with fewer terms
- Model calculation is thus important at this stage
  - Model calculation for TMD is encoraging: See A. Courtoy's talk (July 20)
- Within diquark model, we have found

Kang, Qiu, Zhang, PRD81, 114030 (2010)

  - Soft fermionic correlation functions vanish

$$T_{\Delta q,F}(0,x) = -T_{\Delta q,F}(x,0) = 0 \qquad T_{q,F}(0,x) = T_{q,F}(x,0) = 0$$

  - Only soft gluonic correlation function remains

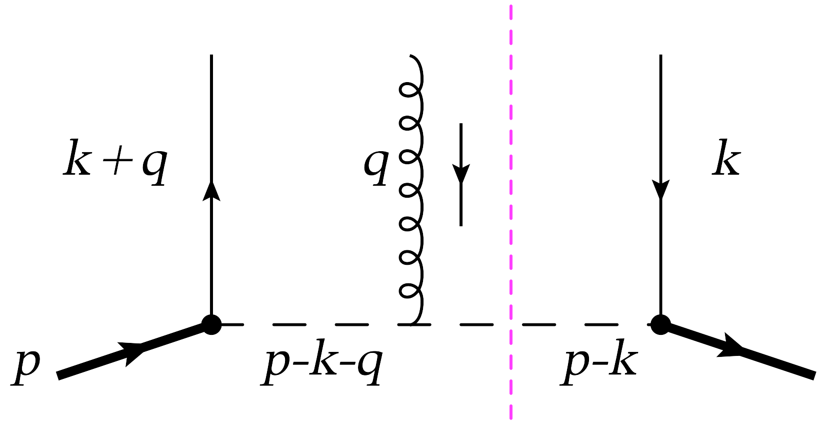
$$T_{q,F}(x,x) \neq 0$$

  - Thus expect soft fermionic contribution small?

Kanazawa, Koike, arXiv:1005.1468

$T_{q,F}(x_1, x_2)$ in diquark model

- Within diquark model, quark-gluon correlation function comes from the following figure



- For soft-fermionic pole case: $(k^+ + q^+) = 0$

$(p-k-q)^2 - M_s^2 + i\epsilon$:

$$q^- = -\frac{1}{2(1-x-y)p^+}\left[\frac{y(k_\perp^2 + M_s^2)}{1-x} + 2k_\perp \cdot q_\perp + q_\perp^2\right] + i\epsilon$$

$q^2 + i\epsilon$:

$$q^- = -\frac{q_\perp^2}{2|y|p^+} + i\epsilon$$

$$(k+q)^2 - m^2 + i\epsilon = -(k_\perp + q_\perp)^2 - m^2 + i\epsilon$$

  - They are all in the upper half plane for q- integral

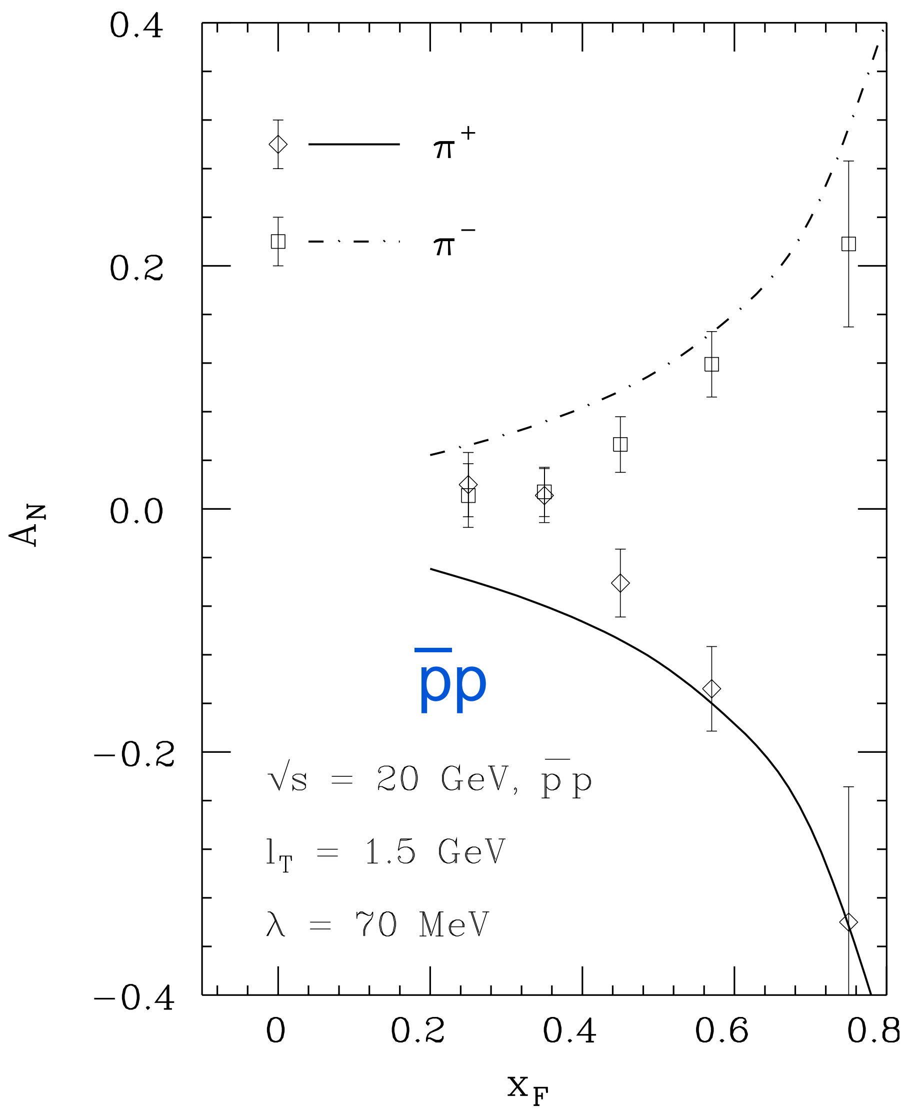
Twist-3 approach: initial success with only $T_{q,F}(x,x)$

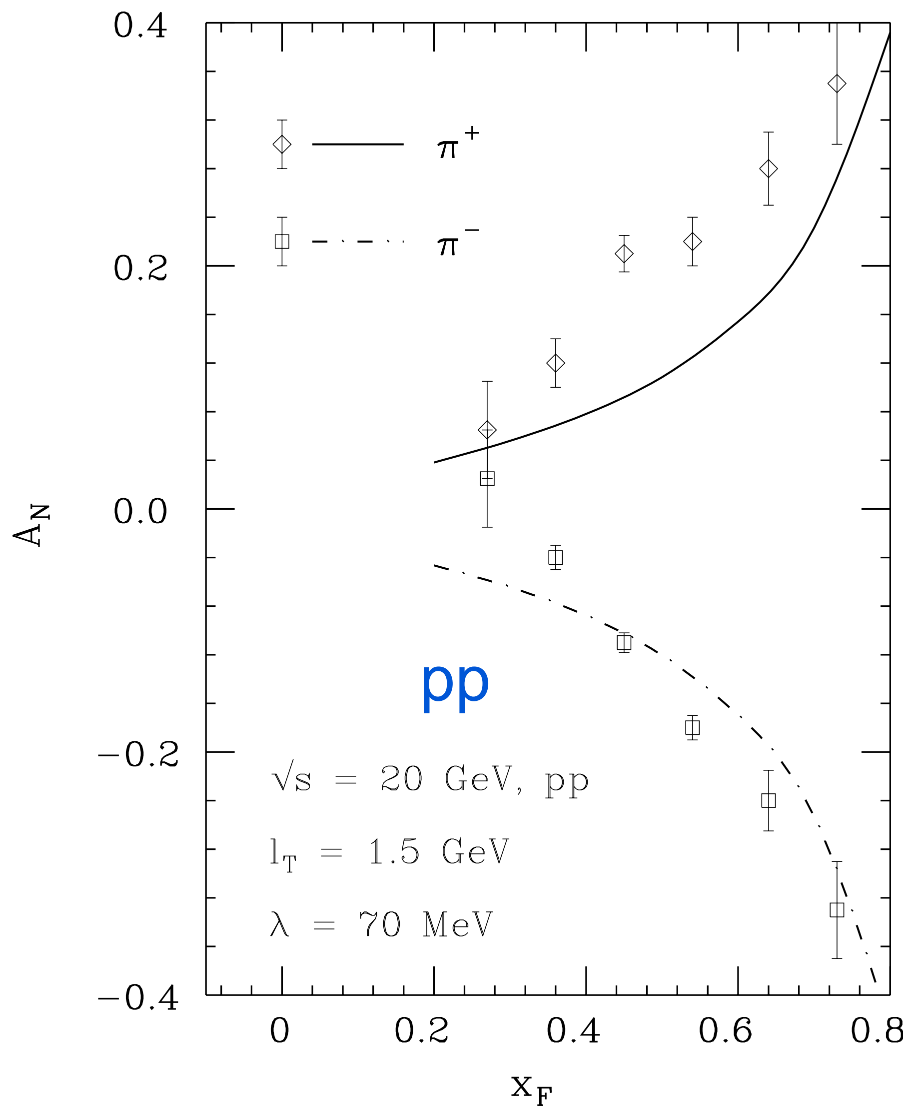
- Describe E704 data well with one parameter (valence quark approx.)

$$T_{u,F}(x,x) = \lambda_F \phi_u(x)$$
$$T_{d,F}(x,x) = -\lambda_F \phi_d(x)$$

$$\lambda_F = 0.07\ \mathrm{GeV}$$

Qiu, Sterman, 1999



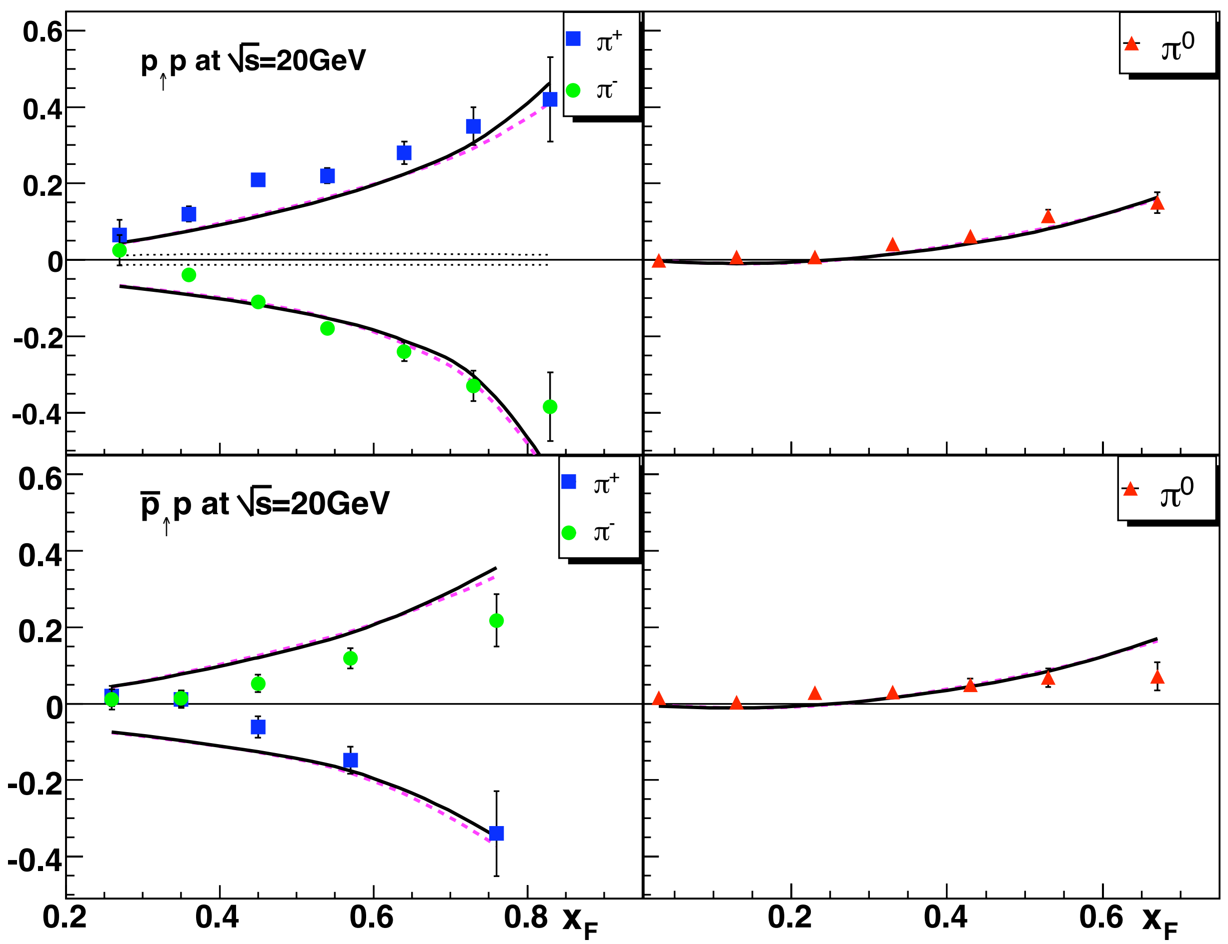


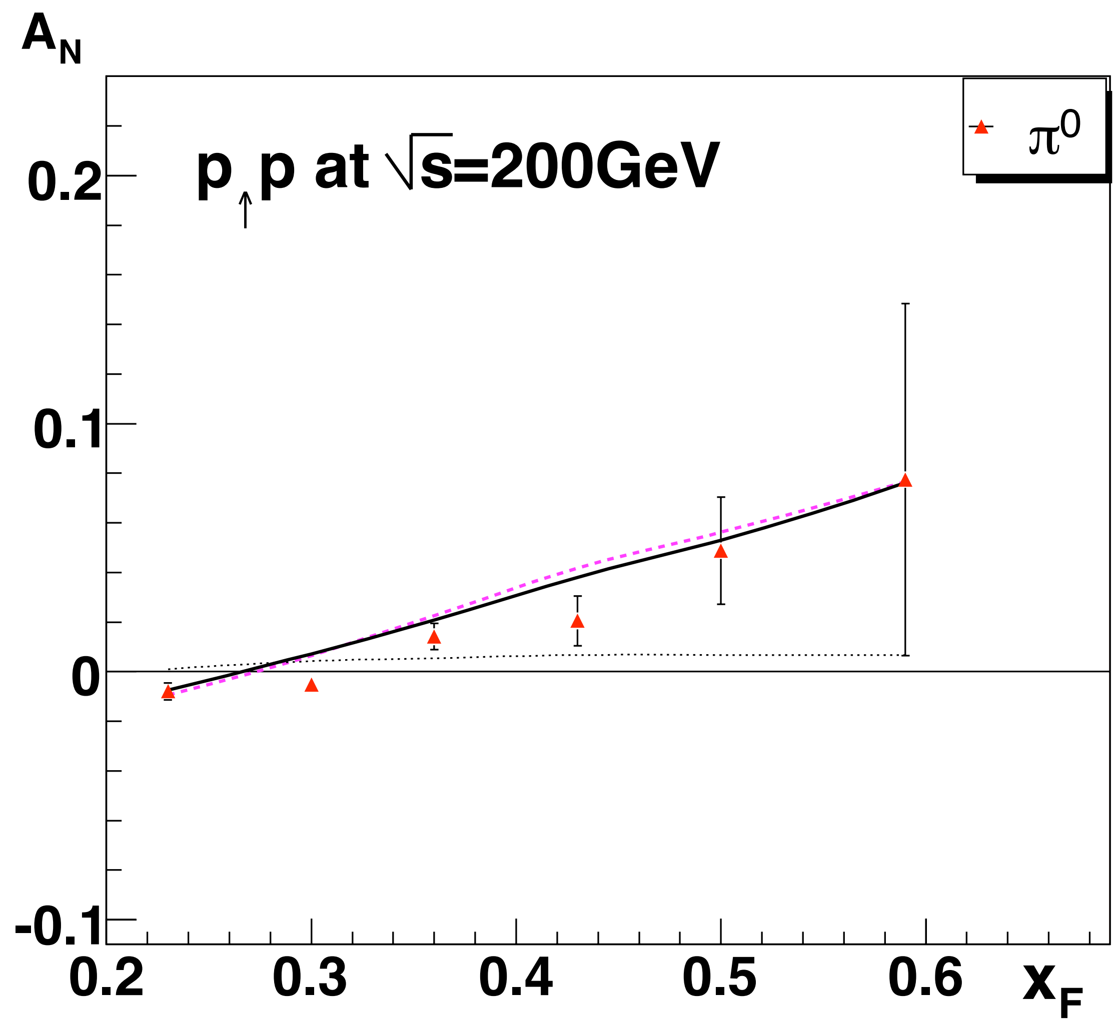
Twist-3 approach: initial success

- Describe both E704 and RHIC data simultaneously with a more sophisticated $T_{q,F}(x,x)$:

Kouvaris, Qiu, Vogelsang, Yuan, 2006

$$T_{q,F}(x,x) = N_q x^{\alpha_q}(1-x)^{\beta_q}\phi_q(x)$$





Besides $T_{q,F}(x,x)$, there are other twist-3 correlation functions.
What about others, particularly gluon?

Twist-3 three-parton correlation functions

- Three-gluon correlations:

$$\mathcal{M}^{\rho\sigma\lambda}(x_1,x_2) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ix_1p^+y_1^- + i(x_2-x_1)p^+y_2} \frac{1}{p^+} \langle p, s_T | F_b^{\rho+}(0) g F_c^{\sigma+}(y_2^-) F_a^{\lambda+}(y_1^-) | p, s_T \rangle$$
$$= \frac{1}{2}\Big[(-g^{\rho\lambda})_\perp \epsilon^{\sigma s_T n \bar{n}} \Big(C^{(f)} \widetilde{T}_G^{(f)}(x_1,x_2) + C^{(d)} \widetilde{T}_G^{(d)}(x_1,x_2) \Big)$$
$$+ (-i\epsilon_\perp^{\rho\lambda})\, i s_T^\sigma \Big(C^{(f)} \widetilde{T}_{\Delta G}^{(f)}(x_1,x_2) + C^{(d)} \widetilde{T}_{\Delta G}^{(d)}(x_1,x_2) \Big) + \cdots \Big]$$

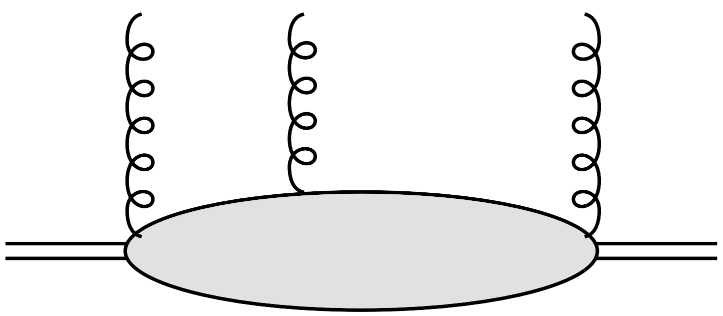
  - two color structures, thus two types of three gluon correlations

$$C^{(f)} = \frac{1}{N_c(N_c^2-1)}(-if_{abc}) \qquad C^{(d)} = \frac{N_c}{(N_c^2-4)(N_c^2-1)}(d_{abc})$$

  - symmetry property:

$$\widetilde{T}_G(x_1,x_2) = \widetilde{T}_G(x_2,x_1)$$

$$\widetilde{T}_{\Delta G}(x_1,x_2) = -\widetilde{T}_{\Delta G}(x_2,x_1) \Rightarrow \widetilde{T}_{\Delta G}(x,x) = 0$$

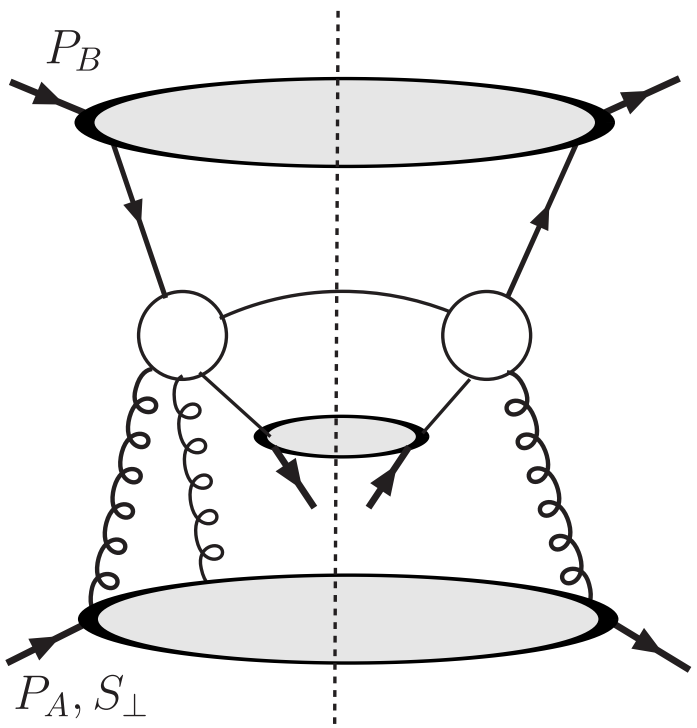


- Relation between $T_G^{(f)}(x,x)$ and $f_{1T}^{\perp g}(x,k_\perp^2)$

$$T_G^{(f)}(x,x) = \int d^2k_\perp \frac{|\vec{k}_\perp|^2}{M} f_{1T}^{\perp g}(x,k_\perp^2)$$

General pattern for gluon channels

- General Feynman diagram:

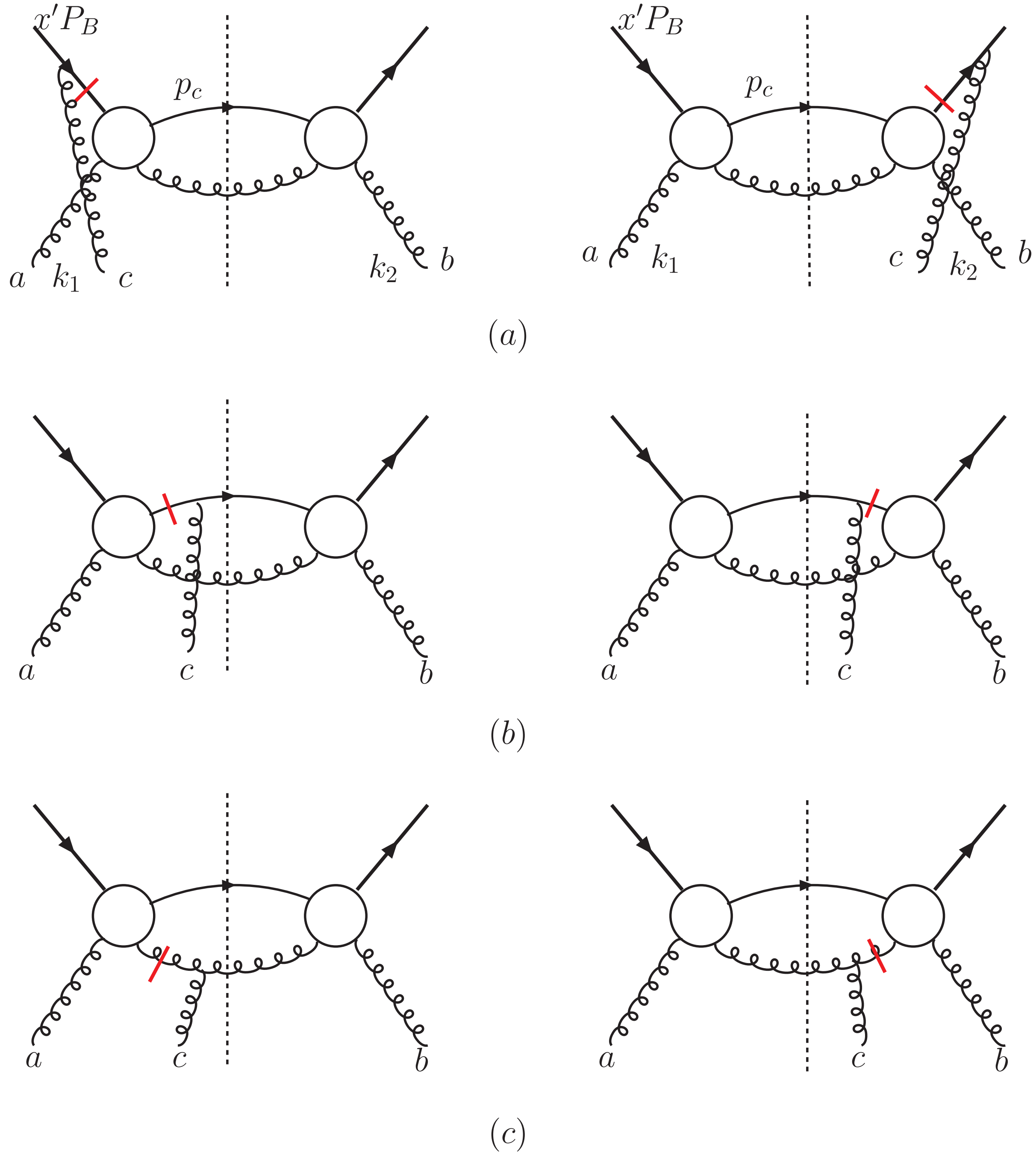


- General factorized form:

$$E_\ell \frac{d\Delta\sigma(S_\perp)}{d^3\ell} = \epsilon^{\alpha\beta} S_\perp^\alpha \ell_\perp^\beta \left[T_G^{(f)}(x,x) \otimes H_{gb\to c}^{(f)} + T_G^{(d)}(x,x) \otimes H_{gb\to c}^{(d)}\right] \otimes \phi_b(x') \otimes D_{c\to h}(z)$$

Typical diagrams

- qg -> qg channel



- Similar for gg -> qqbar, gg -> gg channels

Final results for three-gluon contributions

- Factorized formula:

$$E_\ell \frac{d\Delta\sigma(S_\perp)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_b(x')\frac{\epsilon^{\alpha\beta}S_\perp^\alpha \ell_\perp^\beta}{z(-\hat{u})}$$
$$\times\left[x\frac{\partial}{\partial x}\left(\frac{T_G^{(f)}(x,x)}{x}\right)H_{gb\to c}^{(f)} + x\frac{\partial}{\partial x}\left(\frac{T_G^{(d)}(x,x)}{x}\right)H_{gb\to c}^{(d)}\right]$$

- Hard parts for qg -> qg channel: $H_{gb\to c} = H_{gb\to c}^I + H_{gb\to c}^F\left(1+\frac{\hat{u}}{\hat{t}}\right)$

$$H_{gq\to q}^{(f)I} = \left(-\frac{1}{N_c^2-1}\right)\frac{2(-\hat{t})(\hat{s}^2+\hat{t}^2)}{\hat{s}\hat{u}^2} + \frac{2}{N_c^2(N_c^2-1)}\left[\frac{\hat{s}}{-\hat{t}}+\frac{-\hat{t}}{\hat{s}}\right],$$
$$H_{gq\to q}^{(d)I} = -H_{gq\to q}^{(f)I},$$
$$H_{gq\to g}^{(f,d)I} = H_{gq\to q}^{(f,d)I}(\hat{t}\leftrightarrow\hat{u}),$$
$$H_{gq\to q}^{(f)F} = \frac{1}{N_c^2-1}\frac{2\hat{s}(\hat{s}^2+\hat{t}^2)}{(-\hat{t})\hat{u}^2} - \frac{2}{N_c^2(N_c^2-1)}\left[\frac{\hat{s}}{-\hat{t}}+\frac{-\hat{t}}{\hat{s}}\right],$$
$$H_{gq\to q}^{(d)F} = H_{gq\to q}^{(f)F},$$
$$H_{gq\to g}^{(f)F} = \frac{1}{N_c^2-1}\frac{2(\hat{s}^2+\hat{u}^2)^2}{\hat{t}^2\hat{s}(-\hat{u})},$$
$$H_{gq\to g}^{(d)F} = \frac{1}{N_c^2-1}\frac{2(\hat{s}-\hat{u})(\hat{s}^2+\hat{u}^2)}{\hat{s}\hat{t}\hat{u}}$$

Physics relevant to RHIC spin program

- Gluon's role on generating SSAs
  - Many other processes also receive contribution from tri-gluon correlations

- Single inclusive hadron:	$p^\uparrow p \to \pi + X$ or D meson
- Single jet production:	$p^\uparrow p \to jet + X$
- Direct photon:	$p^\uparrow p \to \gamma + X$
- J/Ψ production:	$p^\uparrow p \to J/\psi + X$
- Drell-Yan:	$p^\uparrow p \to [\gamma^* \to \ell\bar{\ell}] + X$

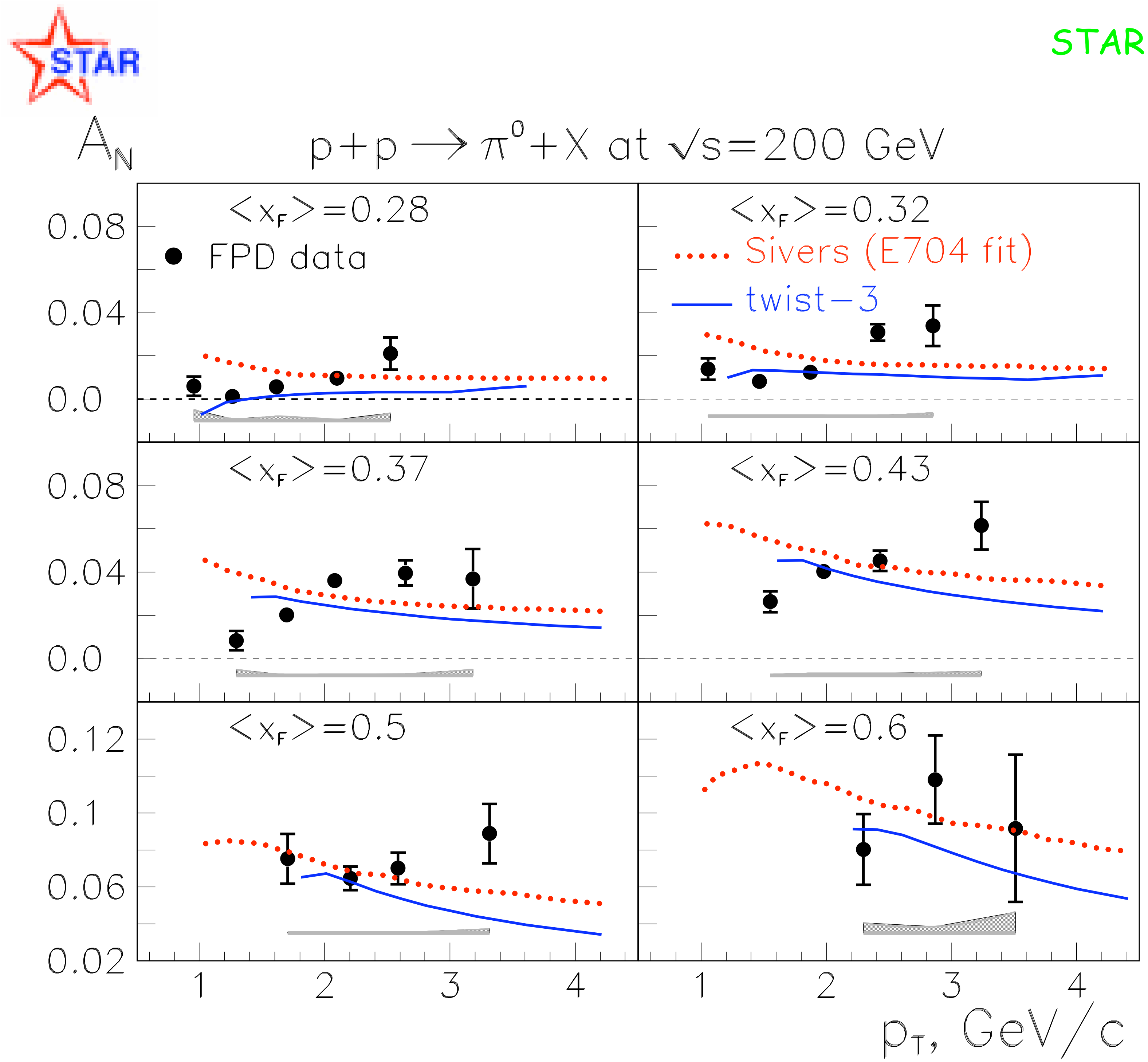
- Global fitting with both $T_{q,F}(x,x)$ and $T_G(x,x)$ included
  - Comparing theoretical SSAs with the experimental data from:

    - E704
    - STAR
    - PHENIX
  - Extract first ever information on $T_G(x,x)$ (also update $T_{q,F}(x,x)$)

New surprise from experiments

- pT dependence of asymmetry for pion production: puzzle?

STAR, PRL 101, 222001 (2008)



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- A_N behavior from low pT to high pT

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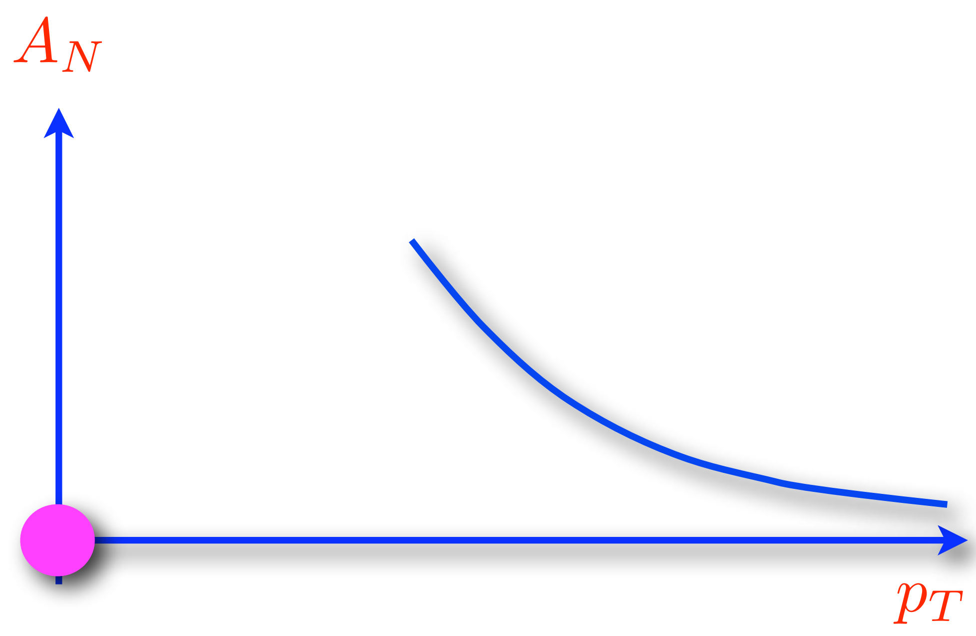
- A_N behavior from low pT to high pT

- pT=0: A_N=0

no plane any more

What should A_N look like?

- A_N behavior from low pT to high pT



- pT=0: A_N=0

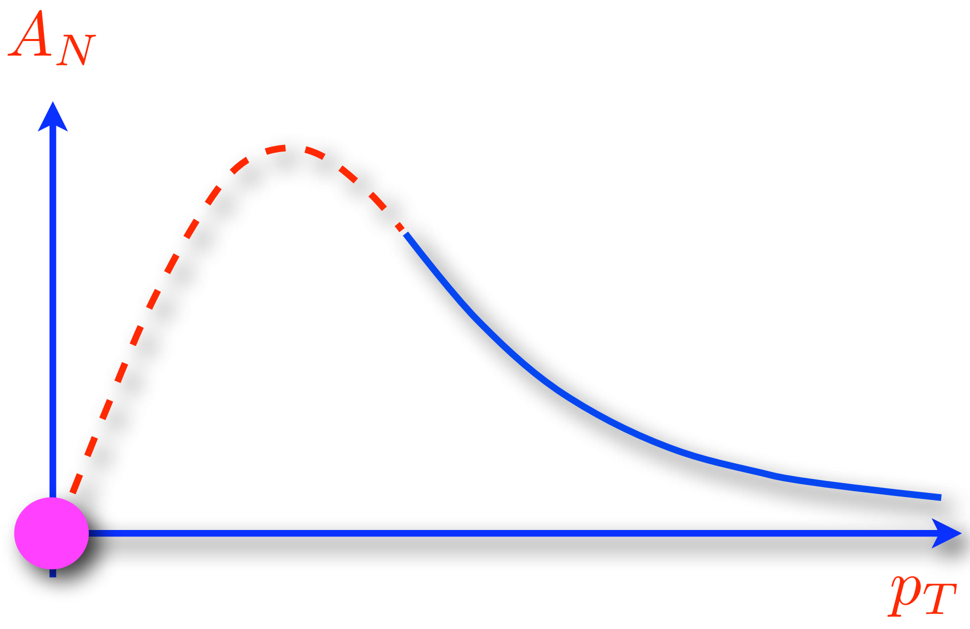
  no plane any more

- very large pT: approach to 0

  higher-twist, suppressed by $1/p_T$

What should A_N look like?

- A_N behavior from low pT to high pT



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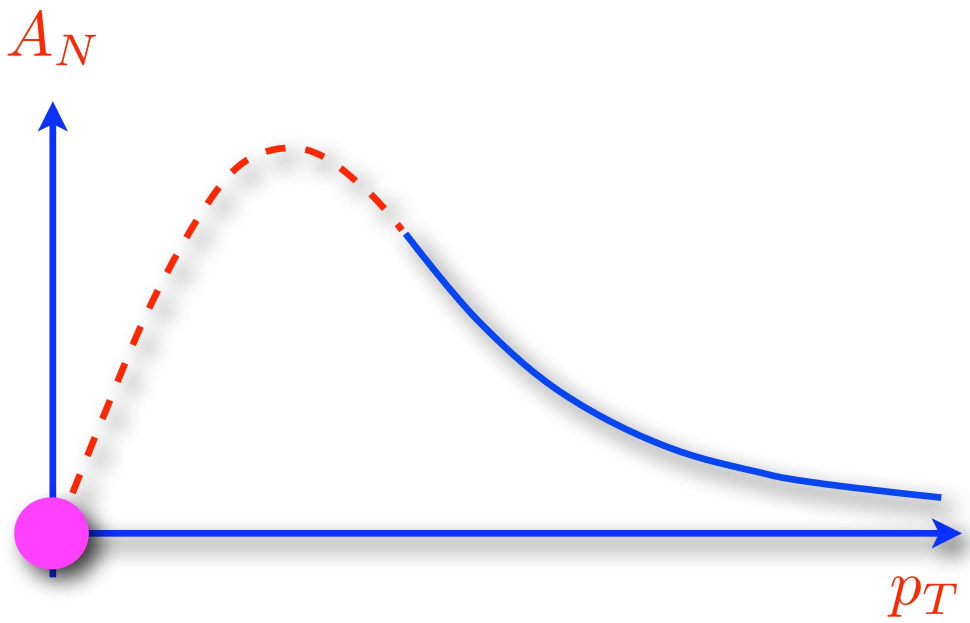


- pT=0: A_N=0
  no plane any more
- very large pT: approach to 0
  higher-twist, suppressed by $1/p_T$

- Natural connection: all power resummation?

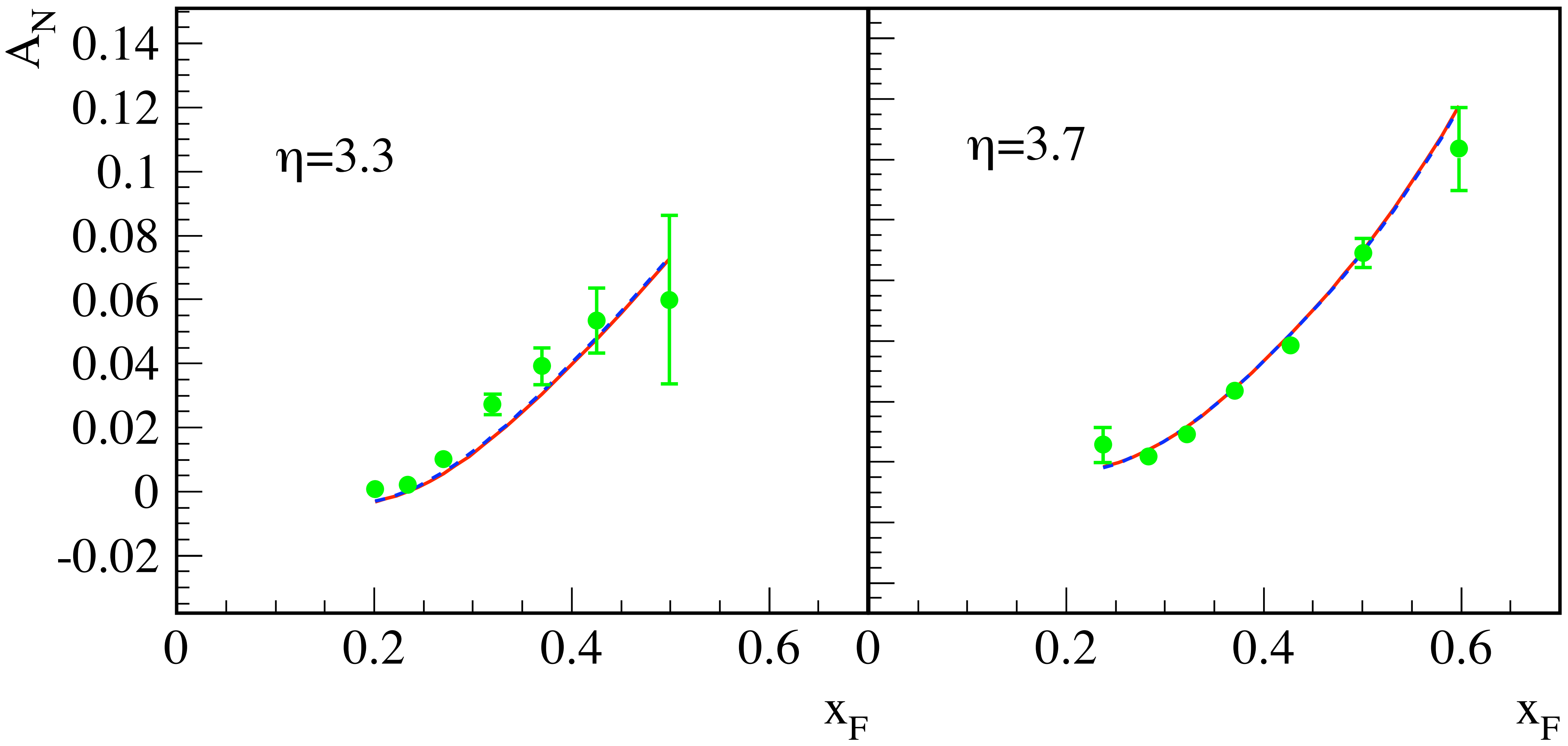
$$A_N \approx \frac{\alpha}{p_T} - \frac{\alpha'}{p_T^3} + \cdots = \frac{\alpha}{p_T}\left(1 - \frac{\Delta^2}{p_T^2} + \cdots\right) \approx \frac{\alpha}{p_T}\frac{1}{1+\frac{\Delta^2}{p_T^2}} = \alpha \cdot \frac{p_T}{p_T^2+\Delta^2}$$

$$\Delta^2 = \frac{\alpha'}{\alpha}$$

$$\boxed{A_N:\quad \frac{1}{p_T} \quad \Rightarrow \quad \frac{p_T}{p_T^2+\Delta^2}}$$

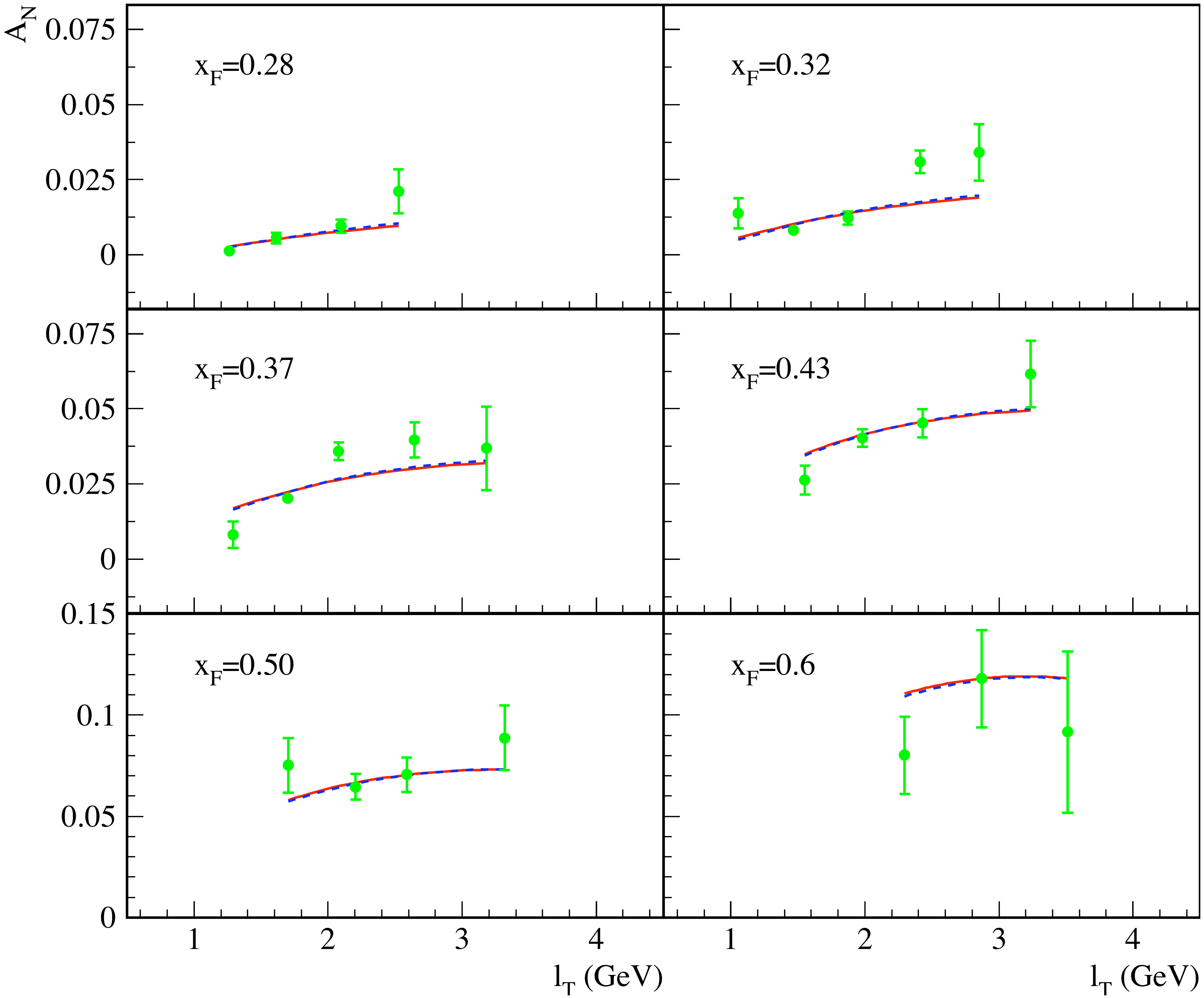
x_F behavior of the SSA

- Two fits: red solid without $T_G(x, x)$, blue dashed with $T_G(x, x)$



New fitting with a new parameter Δ

- New fitting compared with STAR data:



Two fits:

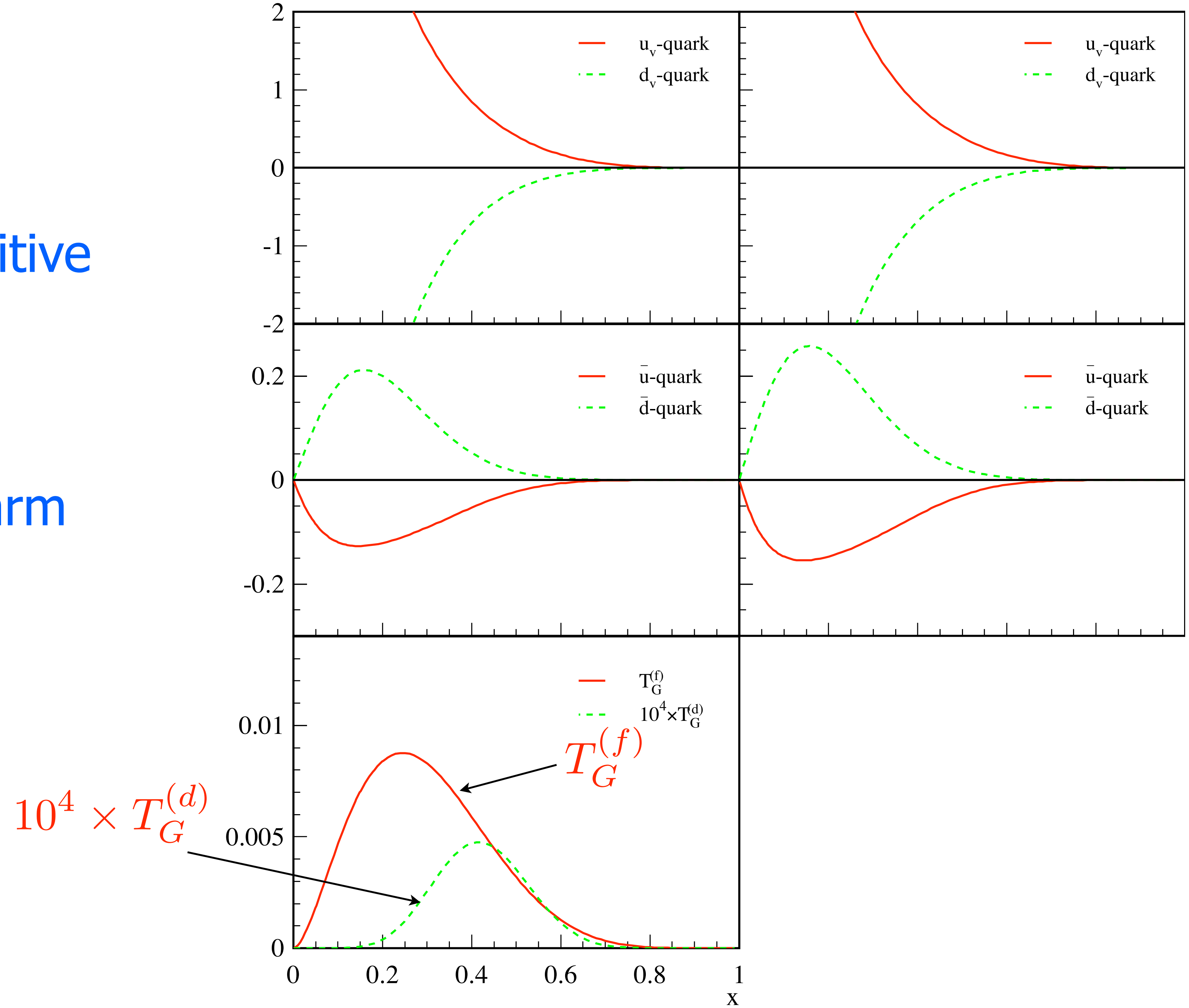
Red solid: without $T_G(x, x)$

Blud dashed: with $T_G(x, x)$

Δ ~ 3 GeV

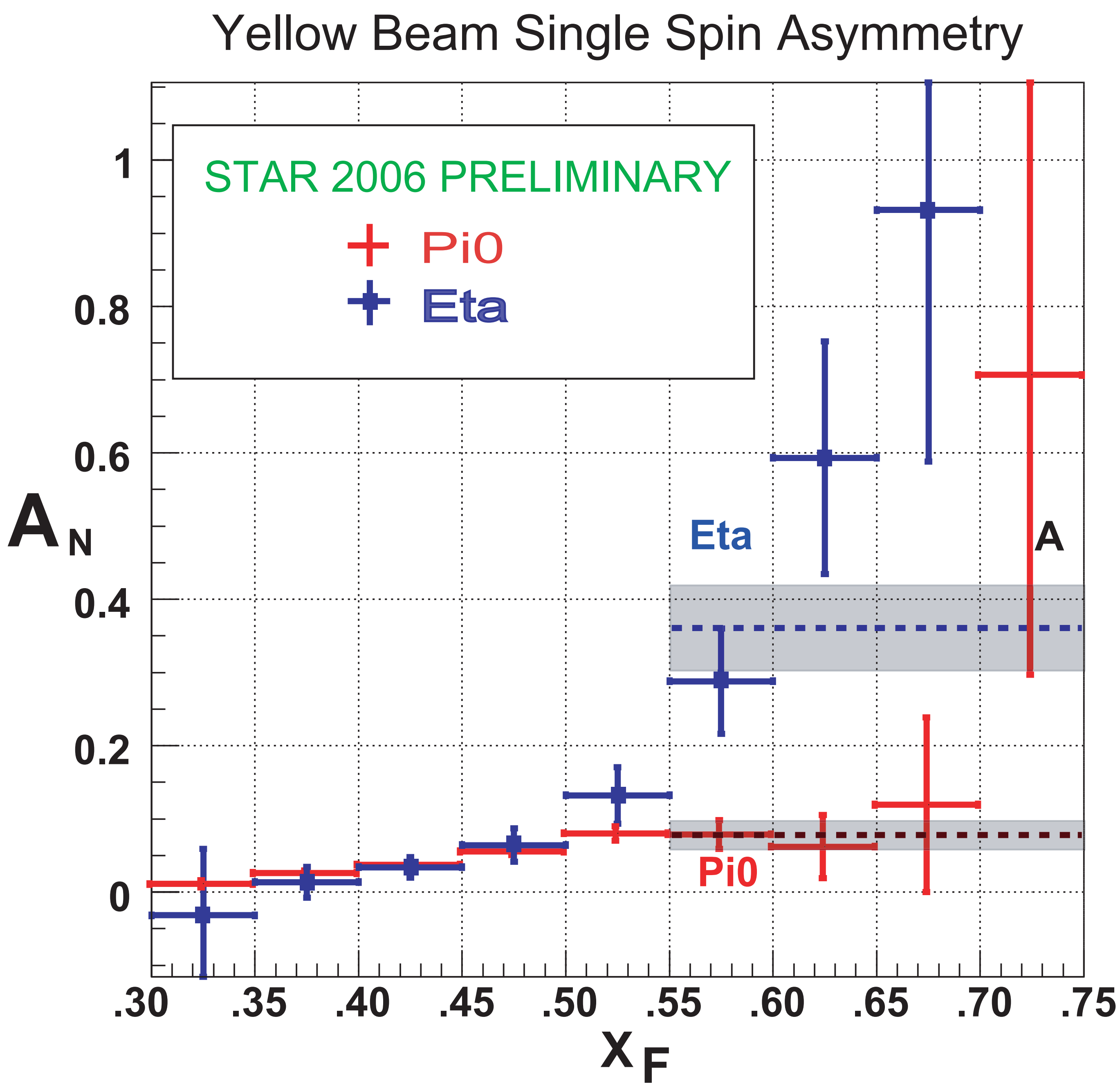
Three-gluon correlation functions

- Seems three-gluon correlation functions are pretty small
  - $T_G^{(f)}$ is small
  - $T_G^{(d)}$ is even smaller
- Pion is not very sensitive to gluon
- Still hoping open charm



Another surprise from experiments: eta meson

- SSA of η meson is much larger than π^0: STAR, arXiv: 0905.2840
  - So far, η meson has looked like a "high-mass, low-yield π^0"



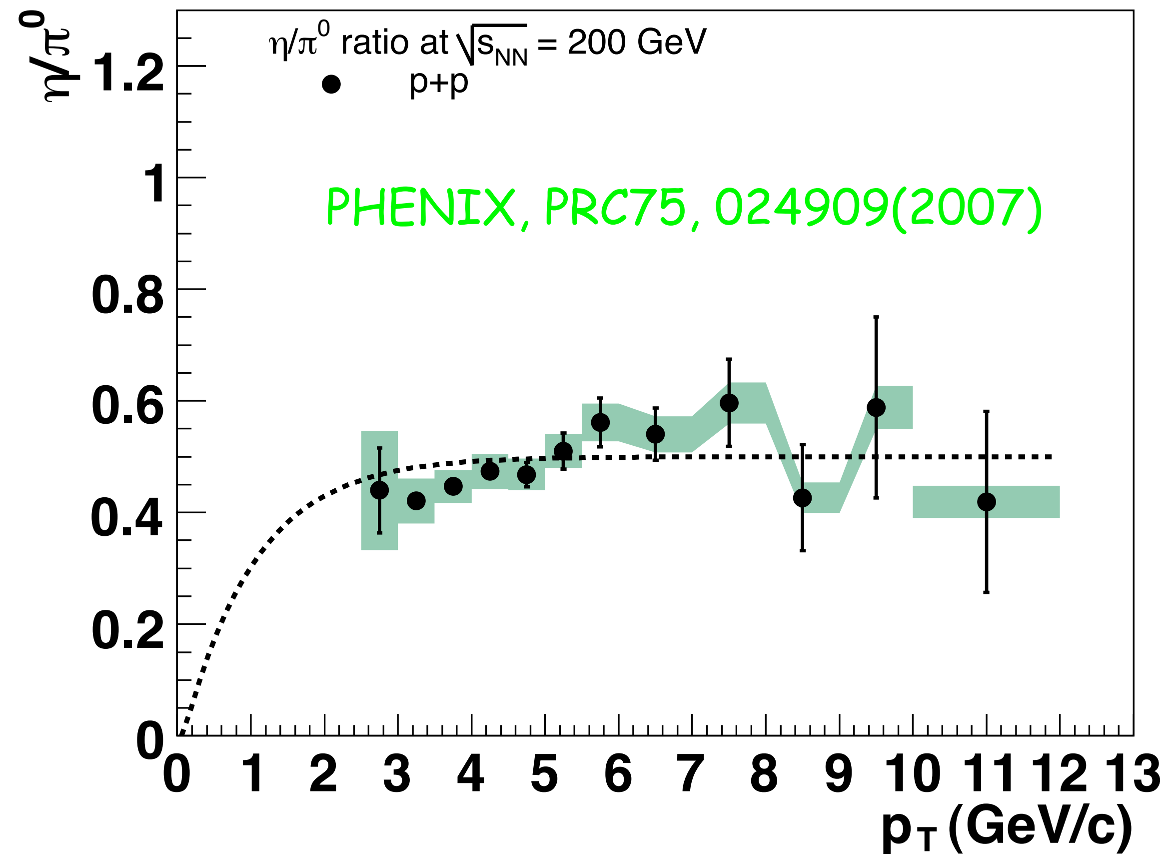
How should we solve this puzzle?

- Could it be described in twist-3 from PDF?
  - Unlikely

$$A_N \propto T_{a,F} \otimes \phi_{b/B} \otimes H_{ab\to c} \otimes D_{c\to\pi^0}$$

The only difference is coming from

- unpolarized fragmentation function for π^0 and η, which is similar



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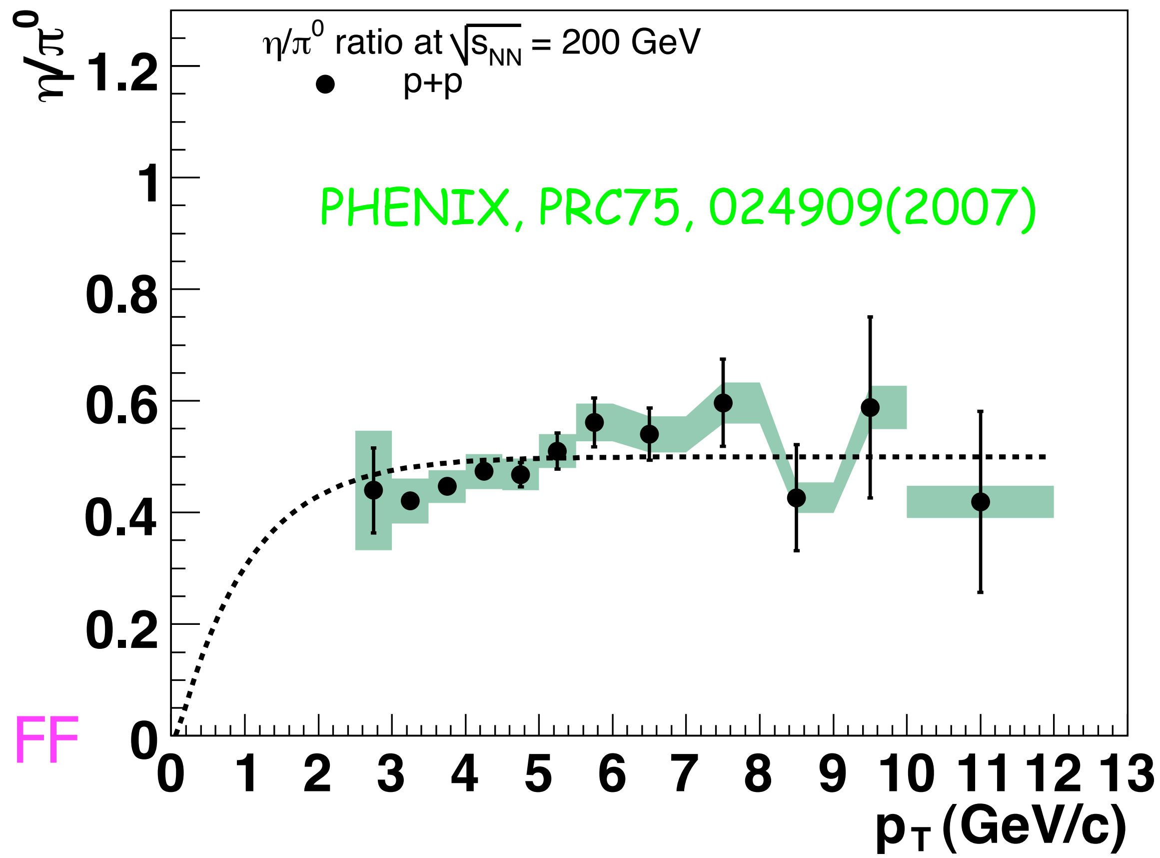

- Collinear version of Collins effect
  - correct twist-3 correlation function from FF

    Yuan, Zhou, PRL103, 052001 (2009)
  - Collins effect for single inclusive meson production in pp collisions

$$A_N \propto \delta\phi_{a/A} \otimes \phi_{b/B} \otimes H_{ab\to c} \otimes \hat{H}_{c\to\pi^0}$$

  - Hard parts $H_{ab\to c}$: Kang, Yuan, Zhou, PLB 2010, in press
  - Model calculation of $\hat{H}_{c\to\pi^0}$: Bacchetta, Gamberg, in progress

Twist-3 fragmentation contribution

- Definition of twist-3 correlation function

$$\hat{H}(z) = \frac{z^2}{2}\int \frac{d\xi^-}{2\pi} e^{ik^+\xi^-} \frac{1}{2}\left\{ \mathrm{Tr}\sigma^{\alpha+}\langle 0| \left[iD_T^\alpha + \int_{\xi^-}^{+\infty} d\zeta^- gF^{\alpha+}(\zeta^-)\right]\psi(\xi)|P_h X\rangle \right.$$
$$\left. \times \langle P_h X|\bar{\psi}(0)|0\rangle + h.c.\right\}$$

  - Related to Collins function:

$$\hat{H}(z) = \int d^2 p_T \frac{|\vec{p}_T|^2}{2M_h} H_1^\perp(z, p_T^2)$$

- Derivative piece only so far:

$$E_h \frac{d^3\Delta\sigma(S_\perp)}{d^3P_h} = \epsilon_{\perp\alpha\beta} S_\perp^\alpha \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{min}}^1 \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x) \int_{z_{min}}^1 \frac{dz}{z}\left[-z\frac{\partial}{\partial z}\left(\frac{\hat{H}(z)}{z^2}\right)\right]$$
$$\times \frac{1}{x'S + T/z}\left(\frac{P_h^\beta}{z}\right) \frac{x - x'}{x(-\hat{u}) + x'(-\hat{t})} H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

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Kang, Koike, Yuan, Zhou, hope to work out complete piece

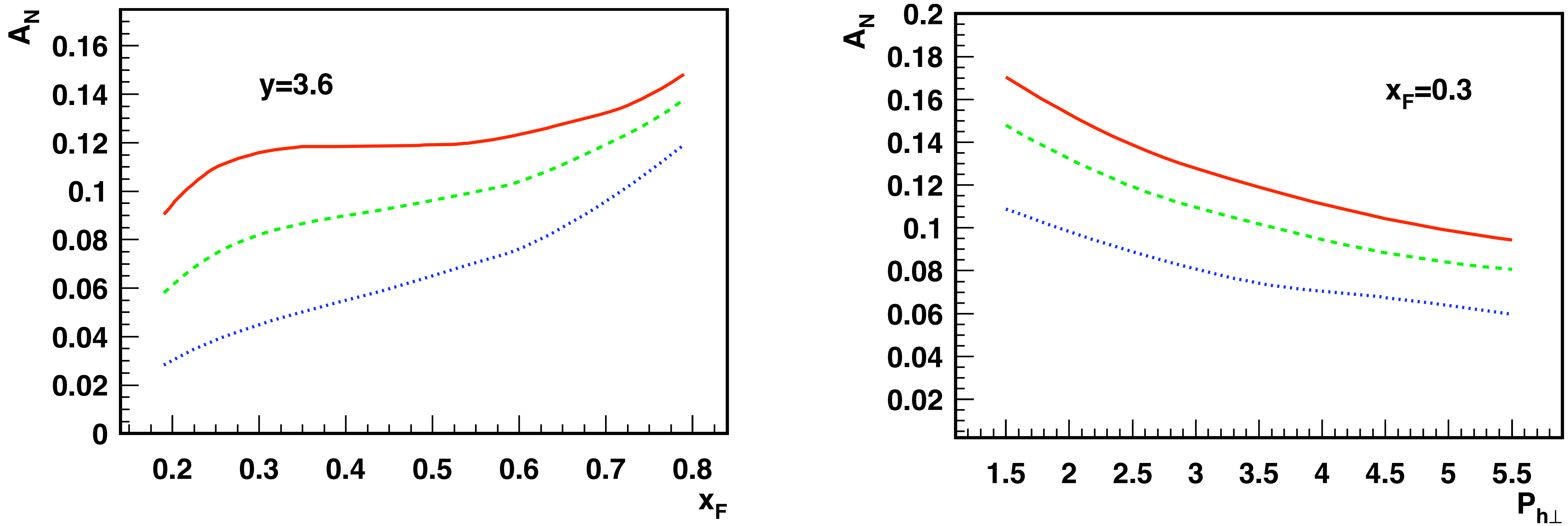
Some rough predictions

- Model for H(z):

$$\hat{H}(z) = C_f z^a (1-z)^b D(z)$$

  - b=0 from power counting arguments at z->1
  - a=1, 2, 4 with C_f=-0.4

- Estimate for pi0 production



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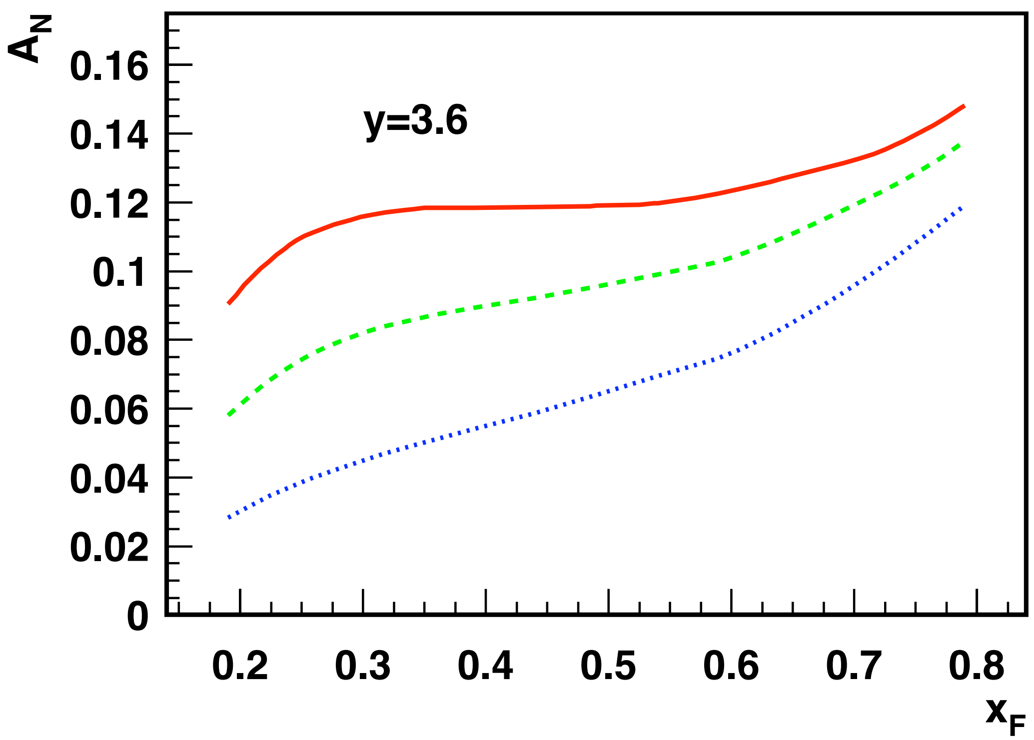

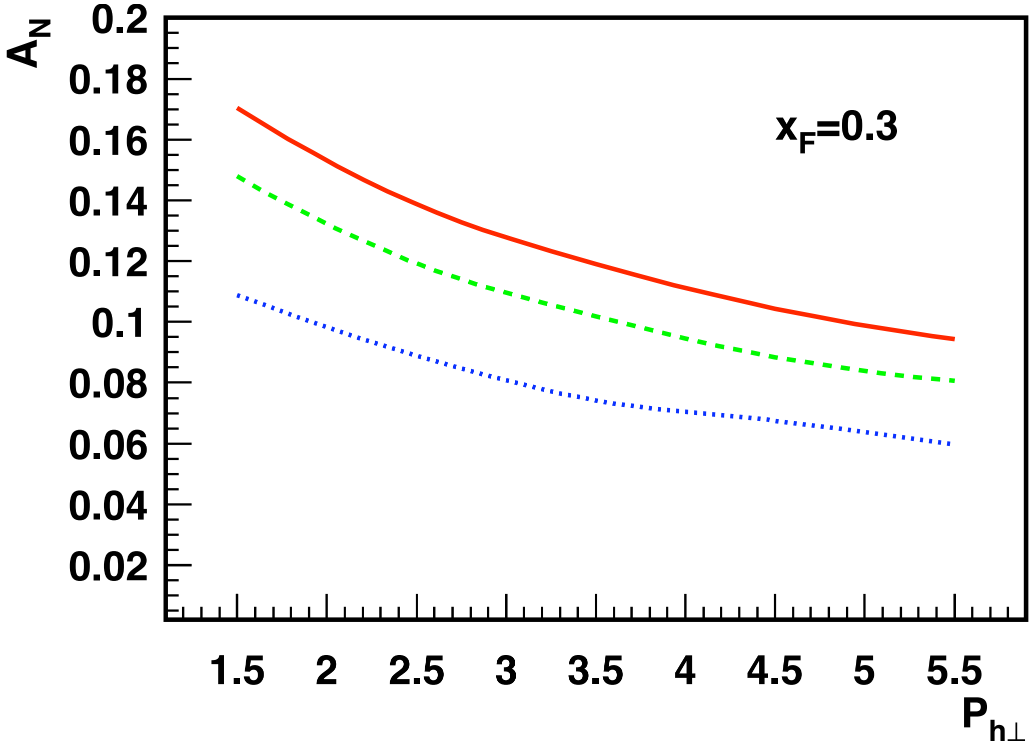

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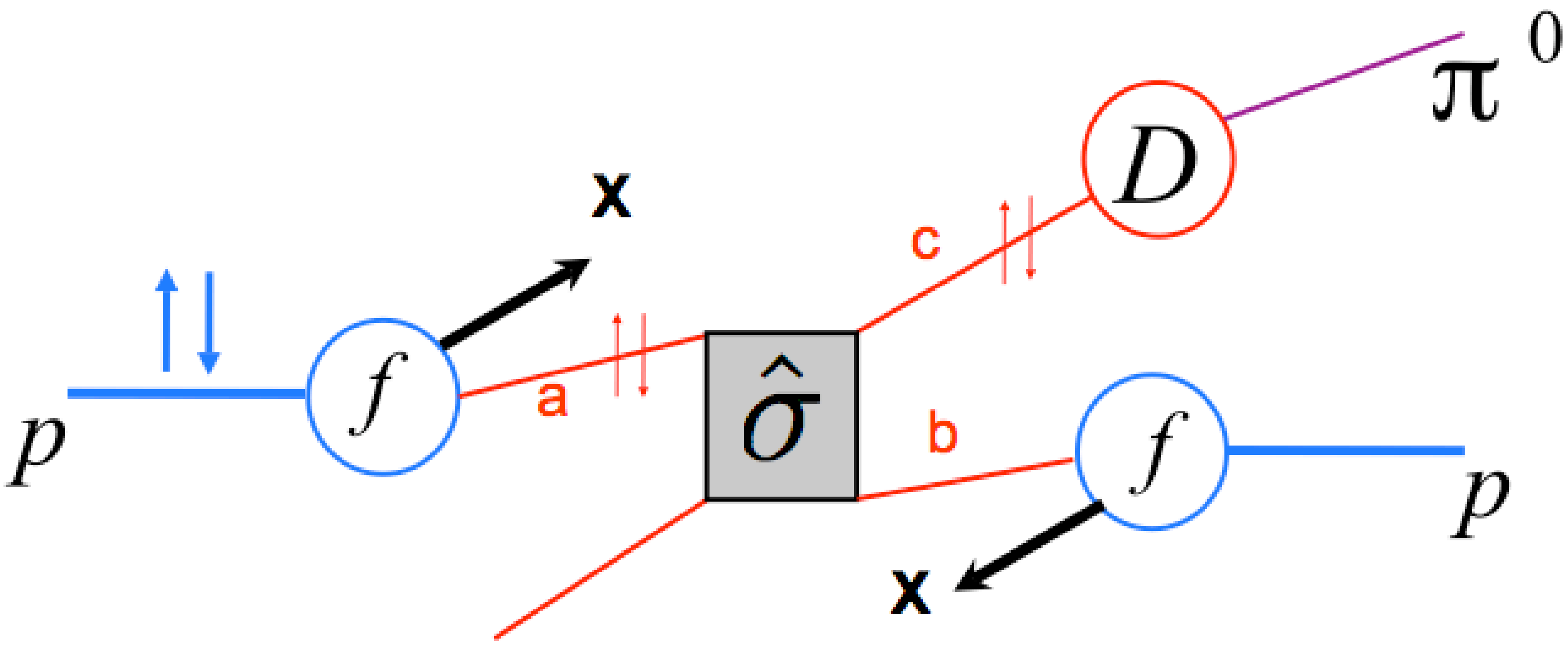
TMD approach for inclusive hadron production

- Generalized Parton Model (GPM) approach

(assuming factorization)



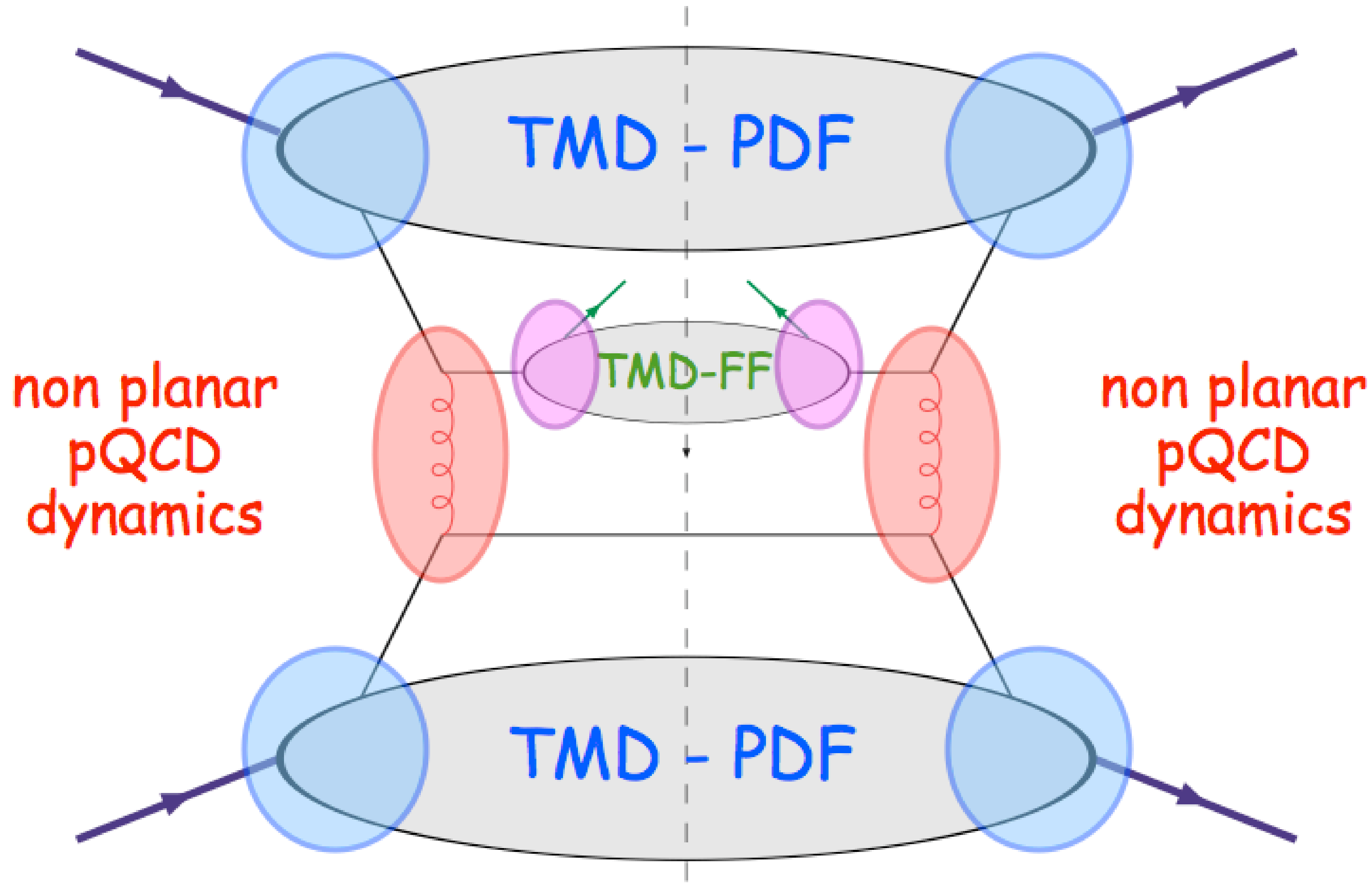
$$d\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes d\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp\pi})}$$

single spin effects in TMDs

M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...
(first proposed by Field-Feynman in unpolarized case)

General diagram for the effects

- Assume a TMD factorization is valid:



Sivers effect in the GPM approach

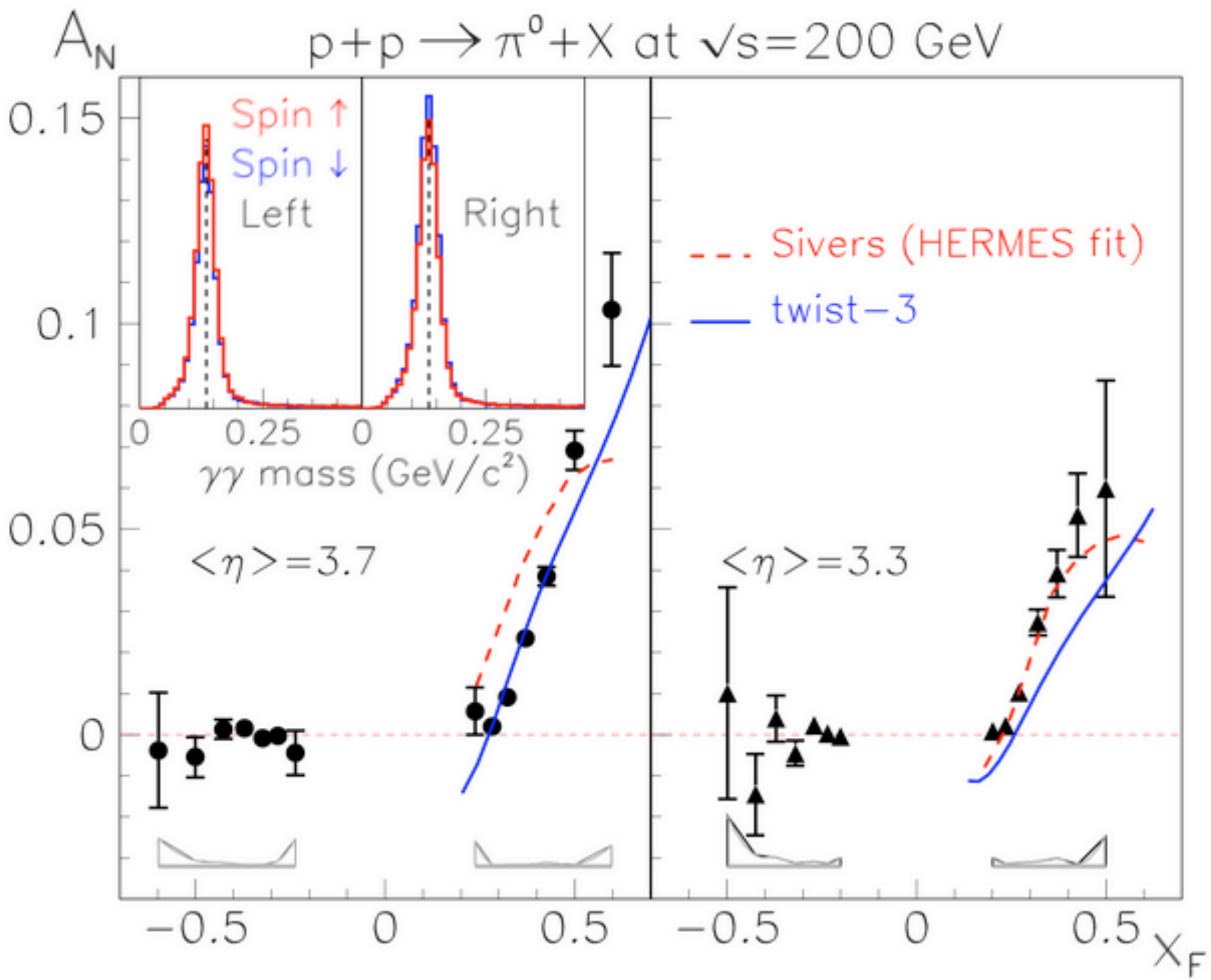
- The general formalism:
  - Spin-dependent cross section:

$$d\Delta\sigma \propto f_{1T}^{\perp}(x_a, k_{aT}) \otimes f_{b/B}(x_b, k_{bT}) \otimes H^U_{ab\to c} \otimes D_{h/c}(z_c, p_T)$$

  - Spin-averaged cross section:

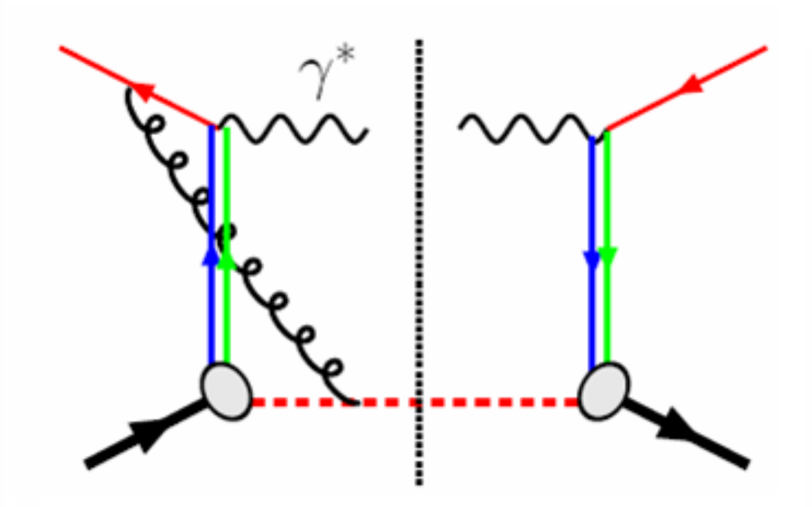
$$d\sigma \propto f_{a/A}(x_a, k_{aT}) \otimes f_{b/B}(x_b, k_{bT}) \otimes H^U_{ab\to c} \otimes D_{h/c}(z_c, p_T)$$

- Use Sivers function extracted in SIDIS, one could make some reasonable description of the RHIC data



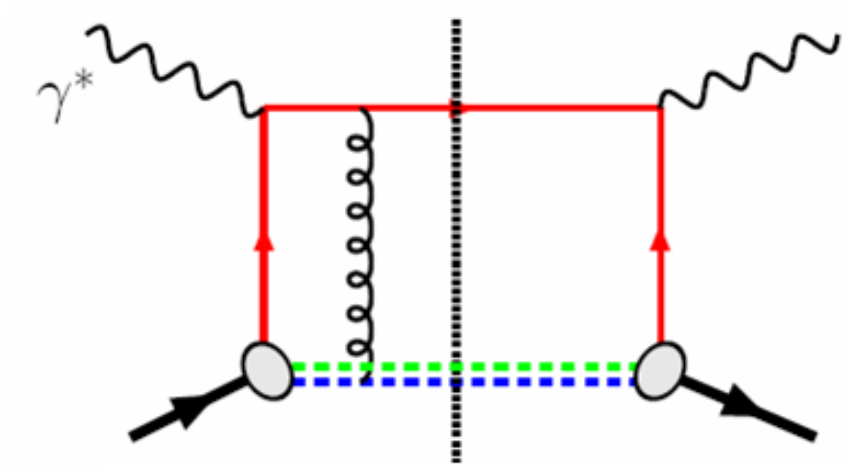
Sivers function needs initial and final state interaction

- Initial- and final-state interaction is very important for Sivers function, which provides the necessary phase to have non-vanishing Sivers function
- Difference between initial and final state interactions



$$p^{\uparrow} + p \to \left[\gamma^* \to \ell^+ \ell^-\right] + X$$

DY: repulsive



$$\ell + p^{\uparrow} \to \ell + \pi + X$$

SIDIS: attractive

$$\Delta^N f_{q/h^{\uparrow}}^{\text{SIDIS}}(x, k_\perp) = -\Delta^N f_{q/h^{\uparrow}}^{\text{DY}}(x, k_\perp)$$

Sivers function is not universal

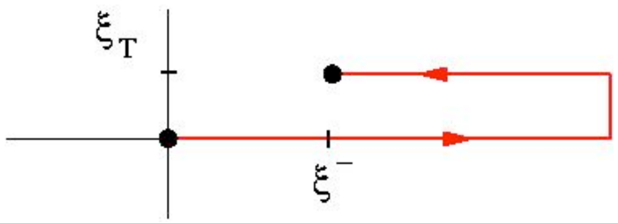
- Initial and final state interaction leads to non-trivial gauge link used to define TMD PDF (or Sivers function)
- Gauge link could be different for different process, thus TMD PDFs are not universal
- Sivers function in inclusive hadron production is different from those measured in SIDIS (or DY)
  - One cannot use Sivers function measured in SIDIS to direct calculate SSA for inclusive hadron production in pp collision as in GPM model

- Question: how to take into account the process-dependence of the Sivers function

Gauge link for different process are derived

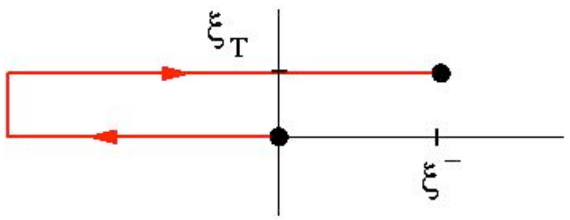
- Gauge link in SIDIS and DY

SIDIS



$\mathcal{U}^{[+]}$

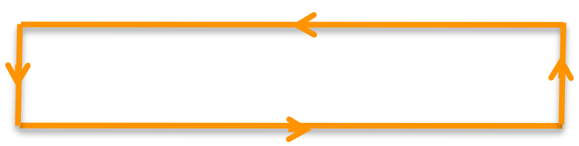
DY



$\mathcal{U}^{[-]}$

- Gauge link in qq′ -> qq′ process:

$$\mathcal{U}_{qq'\to qq'} = \frac{N_c^2+1}{N_c^2-1}\frac{\mathrm{Tr}[\mathcal{U}^{[\square]}]}{N_c}\mathcal{U}^{[+]} - \frac{2}{N_c^2-1}\mathcal{U}^{[\square]}\mathcal{U}^{[+]}$$

$\mathcal{U}^{[\square]} =$ 

- To first non-trivial order (one-glun exchange), one could find:

$$f_{1T}^{\perp}(x,k_\perp)|_{qq'\to qq'} = \frac{N_c^2-5}{N_c^2-1} f_{1T}^{\perp\mathrm{SIDIS}}(x,k_\perp)$$

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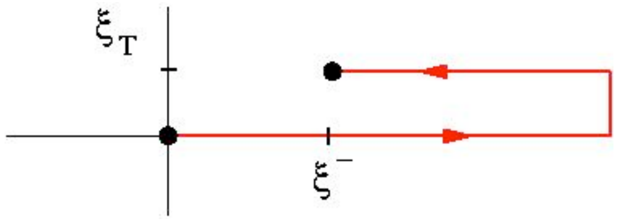

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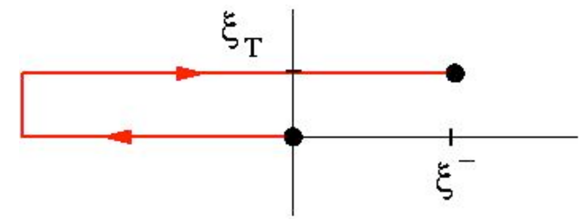
DY



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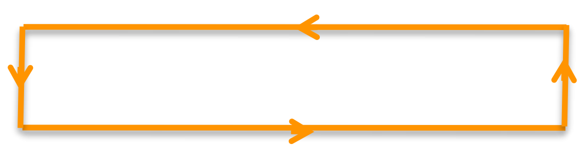

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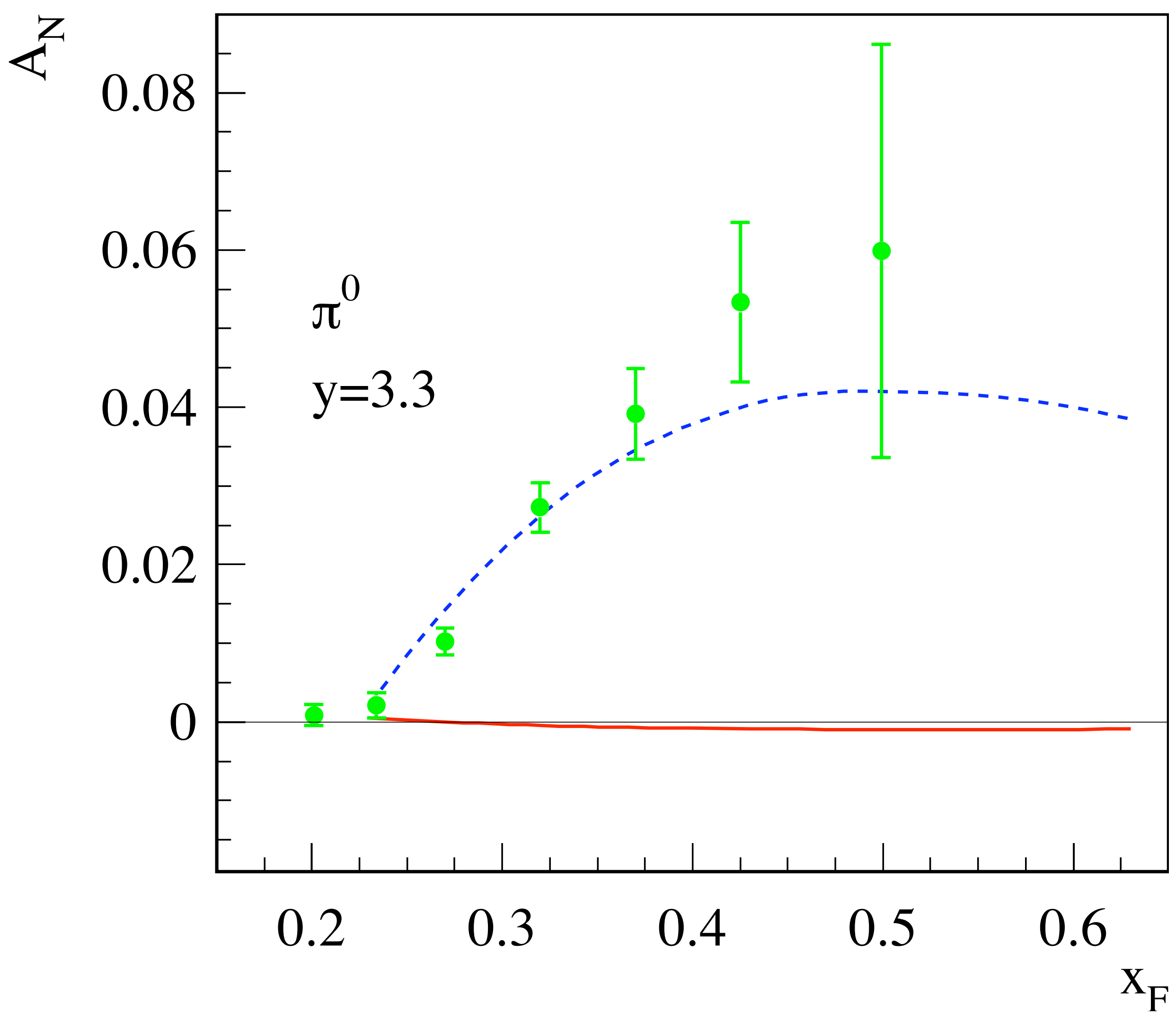
Kang, Gamberg, 2010

Predictions with process-dependent Sivers function

- Do the calculation more consistently: take into account the process-dependence of the Sivers function



Red solid: with process dependence in Sivers function

Blue dashed: without

- If GPM approach is correct, then the SSA for inclusive pion probably does not come from Sivers effect

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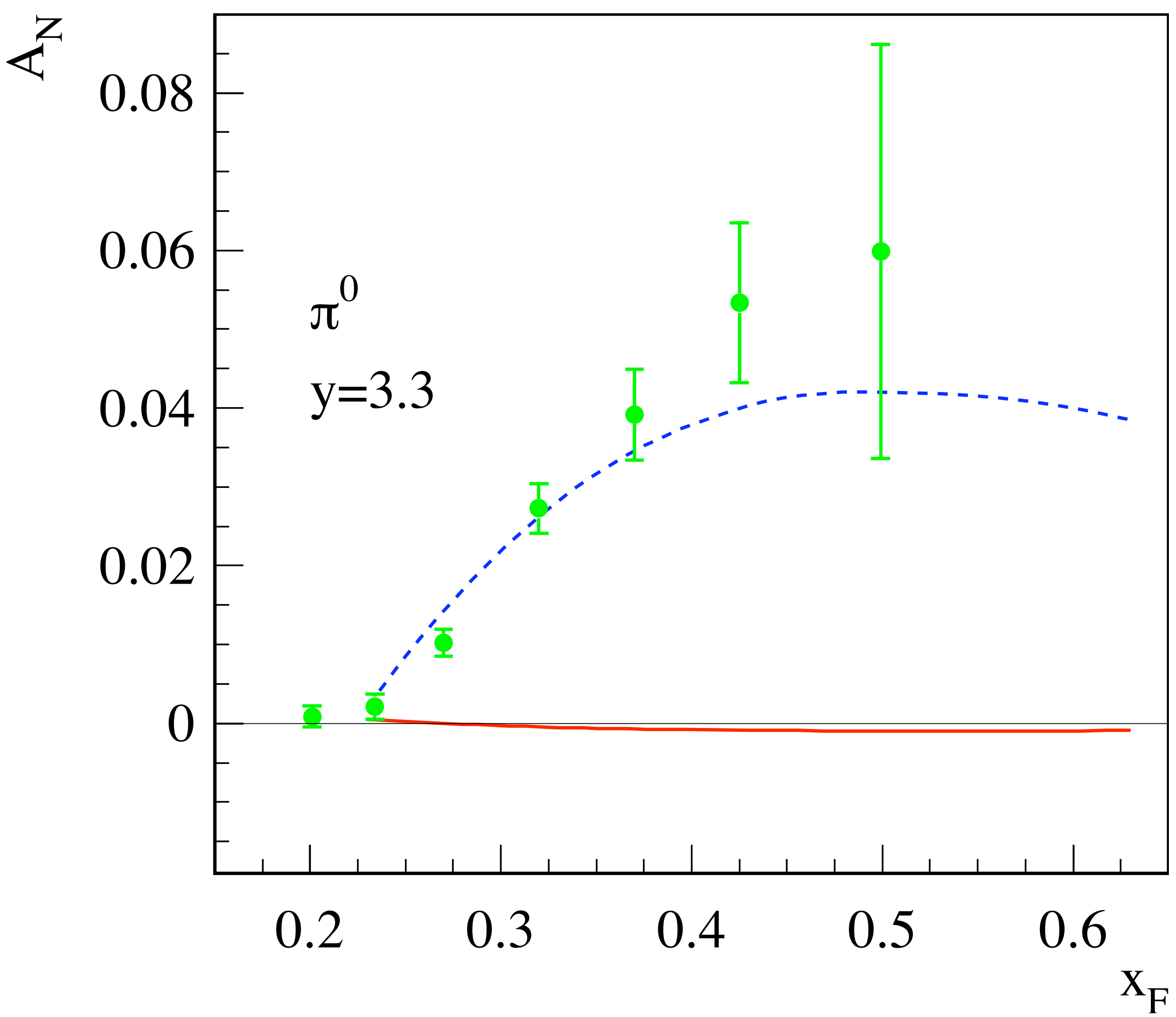

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Summary

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  - an excellent probe for the parton's transverse motion
- More correlation functions (than spin-avg case) are involved, much theoretical progress made for PDF side
  - A better way to describe pT behavior is provided
  - Three-gluon correlation functions are extracted from pion data
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Thank you!

Backup